Working Paper 3.4

A small model for equilibrium mechanisms in an agglomeration

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Revision: 2

FP7-244557

10/03/2012

EUROPEAN COMMISSION
European Research Area
SEVENTH FRAMEWORK PROGRAMME
Funded under Socio-economic Sciences & Humanities

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10/03/2012

Abstract

We use a simple economy with two zones. Individuals can live and work in one of the two zones or can commute between zones. This model is used to explore the dynamics of housing and work decisions following a permanent shock in labor demand in one of the two zones. We illustrate the role of the anticipations of developers and government transport agencies for the equilibrium on the housing and the labor market. The model is used to identify the correct Cost-Benefit rules for transport investments and the role of coordination between housing and transport decisions.

Keywords

Dynamic equilibrium; Transport investments; Residential location; Adaptation

Preferred citation style

de Palma, A., S. Proost and S. van der Loo (2012) A small model for equilibrium mechanisms in an agglomeration, *SustainCity Working Paper*, **3.4**, ENS-Cachan, France.

Executive summary

For an integrated land-use and transport model application the coordination of the actions of the different agents is one of the determinants of the equilibrium. In this paper the role of the equilibrium mechanism has been studied. In urban economics one can distinguish 5 types of markets that are each indexed over time and over space: land market, housing, job, school & amenities, and more generally, transport options.

In each of these markets different equilibrium mechanisms are at work. It is not always clear what mechanism dominates on each of these markets. We distinguish between the following equilibrating mechanisms that can play both at the demand and the supply side (examples):

- equilibration by price (free housing market where households and land developers determine the equilibrium land rent and housing rent for a given location);
- equilibration by price + time and other costs (transport markets where congestion costs and schedule delay costs are part of the equilibrating mechanism);
- rationing (social housing is supplied and allocated at random or using certain eligibility criteria).

In addition one can discuss the availability of future markets and the structure of price and rationing anticipations. There can be a full set of future markets (I can buy a house now but also a house that will be built at a certain spot when I will retire in 15 years) but often there is only a short to medium market (buying houses or signing rental contracts for houses that are available now or will be constructed soon). Anticipations can be forward looking, rational or myopic and within the same category of agents, several anticipation mechanisms can be at work.

A final element of importance is the transaction costs that can be particularly high due to high property transaction taxes.

Current state of the literature

A few papers deal explicitly with this problem but these only consider the housing market (interaction between developers and households). Anas and Arnott (1991&1993) develop a model for one housing market in which they introduce heterogeneity in the housing stock, in consumer tastes and foresee different types of conversions of the housing stock by profit maximizing developers. They propose a perfect foresight equilibrium concept and propose an algorithm to compute it. Martinez and Hurtubia (2006) also propose a land use and housing model in which housing units are unique. Profit maximizing developers supply housing units to the highest bidders. In contrast to Anas and Arnott, they introduce myopic foresight for the developers. Developers are unable to foresee prices correctly and the future prices they anticipate are a weighted average of the present and past prices. Other land use and housing models are less explicit about the supply decisions and the equilibrium mechanism (see e.g. Wegener (2004), Iacono et. al. (2008) for an overview on land-use models and de Palma et. al. (2007)). Still other, more theoretical models only analyze the steady state and its comparative statics properties (see e.g. Arnott and McMillan (2006), Glaeser (2008)).

Current equilibration mechanism in UrbanSimE

In its current set up, there is no standard equilibrium mechanism imposed and the impact of the equilibrium mechanism is hardly documented. We understand that in most applications, UrbanSimE is used as a myopic equilibrium model. The land use planner comes first by identifying the type of use for each plot of land. Next developers can increase the supply of buildings using an anticipation mechanism specified by the user – in general as a function of past prices. Households select the type and location of their house in function of relative rents and the transport costs (via an accessibility indicator) to different types of destinations. The households do in principle not anticipate any future developments. The firms have a similar type of myopic behaviour as the households.

All in all, the model has myopic developers and households. It is only the land use planner (the user of the model) that can anticipate the reaction of the developers and the households and firms.

As far as we know there have been no extensive tests to assess the role of the equilibration mechanisms in UrbanSimE.

Lessons from our research on equilibrium mechanisms

We investigated the role of anticipations in the housing market and the possible interplay with transport investments. We used a simple two region model where one region is subject to an unanticipated productivity shock that increases the demand for labour in that region. The demand for labour can be met by additional commuting, by more housing or by additional investment in transport. When no additional investment in transport is possible, the result depends on the expectations of the developers. Developers with myopic expectations arrive also at the optimal housing stock but need more cycles to reach the optimum.

When we add a transport agency that can invest to improve the transport infrastructure between the two regions there is a coordination problem. Transport investment tend to have longer construction periods and lifetimes than housing investments. In this case the steady state equilibrium will mainly depend on the anticipation of the transport agency. Depending on the anticipations, we can see over- or underinvestments in transport infrastructure.

The model is a simplistic two region model with only one mode of transport. Interesting extensions include the consideration of the introduction of regional governments that can tax income and commuting flows, decide on the building permits etc.

1 Introduction

How cities evolve is the result of complex interactions between households, developers, firms and transport agencies. Describing the behavior of all these agents simultaneously is not obvious and is made even more complex due the different speeds with which the different urban change processes occur. Construction of new transport infrastructure is typically a very slow process; large transport infrastructures, once in place are more or less permanent elements but they can take a very long time to construct. Buildings also have a long life-span and take several years to complete. Population and employment, on the other hand can change fast: firms are established or close down, households formed or dissolved and can move. Transportation itself is the most flexible of all processes and can change almost instantaneously.

The purpose of this paper is to study the dynamics of housing and work decisions following a permanent shock in labor demand. The differences in speed of adjustment imply that decisions will be made on expectations rather than known market prices. The paper focuses especially on the role of these expectations on the future rental prices of the developers and the impact of government decisions on the transport infrastructure.

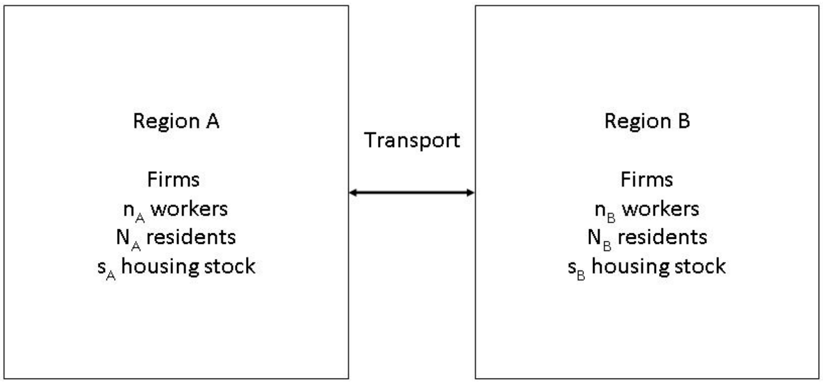
A few papers deal explicitly with this problem but these only consider the housing market (interaction between developers and households). Anas and Arnott (1991&1993) develop a model for one housing market in which they introduce heterogeneity in the housing stock, in consumer tastes and foresee different types of conversions of the housing stock by profit maximizing developers. They propose a perfect foresight equilibrium concept and propose an algorithm to compute it. Martinez & Hurtubia (2006) also propose a land use and housing model in which housing units are unique. Profit maximizing developers supply housing units to the highest bidders. In contrast to Anas and Arnott, they introduce myopic foresight for the developers. Developers are unable to foresee prices correctly and the future prices they anticipate are a weighted average of the present and past prices. Other land use and housing models are less explicit about the supply decisions and the equilibrium mechanism (see e.g. Wegener (2004), Iacono et. al. (2008) for an overview on land-use models and de Palma et. al. (2007)). Still other, more theoretical models only analyze the steady state and its comparative statics properties (see e.g. Arnott and McMillan (2006),Glaeser (2008)).

The paper is organized as follow: first we introduce a simple two zone model which will be used to analyze the reaction of housing, labour and transport market to an exogenous productivity shock in one region. After a sketch of the different components of the model in Section 2, we study the different laissez faire equilibriums without transport investment (Section 3) for three different expectation mechanism (perfect foresight, delayed reaction and myopic expectation). Next we introduce a government that have the option to change the transport infrastructure and study the impact of a reduction of the commuting costs on the housing and labor market (Section 4). In the Section 5 we make some extreme assumptions on the building costs of housing and investment costs to illustrate the findings. The last section concludes.

2 Model components

We consider two regions, labeled A and B. There are N households consisting of a single worker which choose simultaneously their residence and their workplace. Workers are allowed to commute, so that job and residence location need not to be in the same region. In each region there are competitive firms employing the n workers. Each household consumes one unit of housing which is provided by profit maximizing developers. We use Figure 1 to introduce the basic notation.

Figure 1 Elementary notations



We distinguish different periods $t = 0,...T$. At the beginning of each period households choose their workplace and residence location, firms choose how much labor they need and developers decide how many new houses will be built in each region.

2.1 Individuals

Let N_A $\left(N_B\right)$ denote the number of (homogenous) individuals living and working in A (B). The number of commuters, i.e. individuals living in region I but working in region J, is labeled N_{IJ}. We assume that there is no migration so that the number of individuals in the economy is fixed and equal to N. We further assume that all individuals work and that the number of individuals is large enough so that they take wages and house prices as given. Individuals are assumed to jointly select a residential location and a work location so as to maximize utility. The indirect utility of an individual (U) depends on his net income which equals his wage (w) minus his housing costs (r) minus the commuting costs (tt),where the average commuting cost is a linear function of the number of commuters[1]:

$$U_{ij}(t) = U\left(w_j(t), r_i(t), tt_{ij}(t)\right) = w_j(t) - r_i(t) - tt_{ij}(t), \qquad i, j = A, B.$$

$$tt_{ij}(t) = \mu N_{ij}(t).$$

As individuals can relocate at the beginning of every period, a spatial equilibrium for the individuals implies that the utility of every individual will be equal whatever his choice of residence and job. In addition, if there is commuting at equilibrium, the commuters have the same utility as those that do not commute:

$$U_{AA}(t) = U_{BB}(t) = U_{BA}(t), \tag{1}$$

$$w_A(t) - r_A(t) = w_B(t) - r_B(t) = w_A(t) - r_B(t) - \mu N_{BA}(t). \tag{2}$$

Workers will only be willing to commute from B to A if wages in region A are higher to compensate the commuting costs. The differences in rents between the two regions will compensate the wage differences between the two regions.

2.2 Firms

On the production side of the economy we assume that there are firms in each region producing a single homogenous goood and use labor as only input. Contrary to the individuals who can choose their residential location at each period we assume the location of the firms as fixed. In each region all individual firms have the same quasi-linear production function. The aggregate marginal product function is decreasing in the number of workers n (this means that we neglect agglomeration effects which would imply an increase in marginal productivity with the number of workers):

$$MP_i\left(n_i(t)\right) = \alpha_i - \beta_i n_i(t), \qquad i = A, B. \tag{3}$$

[1]We only consider commuting from region B to A, therefor N_{BA} (the number of commuters) can be negative when commuting takes place in the other direction.

Production is sold on the world market and price of the product is normalized to 1. The shareholders of the firms are assumed to be residents of the region where the firm is located, so that profits will benefit the region in which the firm is located. Labor and product are homogeneous and perfect competition on the product and labour market implies that the marginal product is equal to the wage (w):

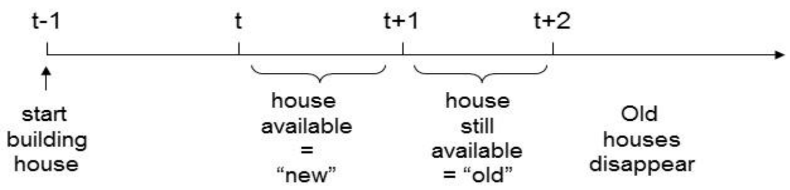
$$MP_i\left(n_i(t)\right) = w_i\left(t\right) \Rightarrow n_i(t) = \frac{\alpha_i - w_i(t)}{\beta_i}, \qquad i = A, B. \tag{4}$$

We will refer to this condition as the labour market equilibrium condition.

2.3 Developers

While households can reallocate every period and firms can adjust their wages without delay, we will assume that building a house does take time. We assume that a housing unit lasts for two periods and it takes one full period to construct a house. Houses build in period $t-1$ and available in period t will be called " new" houses in period t, the same housing units available in period $t+1$ will be called " old" houses (see Figure 2).

Figure 2 Evolution of housing stock over time



There is no depreciation or increased maintenance cost for old housing. We further assume that at the end of each period, individuals can move and relocate at a fixed moving cost M that can be zero.

The housing stock $s(t)$ in each period t is the sum of the houses built $b_i(.)$ in period $t-1$ and $t-2$:

$$s_i(t) = b_i(t-1) + b_i(t-2). \tag{5}$$

The total construction cost (including the cost of land) of new houses $b(t)$ built in period t in zone i is given by

$$TC_i(b_i(t)) = \gamma b_i(t) + \frac{\delta}{2}(b_i(t))^2 \quad \forall t, i = A, B. \tag{6}$$

Houses are rented to residents at a rental price $r_i(t), i = A, B$ at each period t. The rents are determined by demand and supply on the housing market. The developers and households take rents as given (perfect competition assumption).

Developers maximize profits when deciding how much houses to build. Their housing supply maximizes discounted profits in each period (where we use the discount factor $\rho = 1/1+ii$ where ii is the real interest rate for alternative investments on the international capital market):

$$\Pi_i(b(t)) = \rho\{r_i(t+1) + \rho r_i(t+2)\}b(t) - \left(\gamma + \frac{\delta}{2}b_i(t)\right)(b_i(t)). \tag{7}$$

Perfectly competitive developers take the rents as given; they will build until the discounted profit equals the marginal cost and if discounted rents do not cover the marginal construction cost, building activity will decrease and can disappear:

$$\{\rho r_i(t+1) + \rho^2 r_i(t+2)\} \geq \gamma + \delta(b_i^*(t)). \tag{8}$$

We see the role played by future rents in the building decisions of the developers. We have now all the elements to describe demand, supply and equilibrium on the housing market.

The demand for houses equals the equilibrium number of inhabitants in that region. The supply of houses consists of the houses constructed in that region in the two previous periods:

$$N_A(t) = s_A(t) = b_A(t-1) + b_A(t-2). \tag{9}$$

Up to now we have nothing that prevents vacancies or a lack of housing units for the given population. A good treatment of vacancies requires reconversion options for older houses (as in Anas & Arnott) and a shortage requires the reconversion of housing space into smaller units. In this exploratory phase we avoid these complexities and introduce an artefact to deal with both problems. We put a minimum rental price in the market by assuming that at a rent r_{MIN} there is an infinite demand for housing units for another purpose (commercial storage, ..).

And we use a maximum price r_{MAX} at which there is an infinite and immediate supply of temporary housing (containers, ..).

2.4 The transport agency

Since the purpose of the paper is to understand the interaction between land developers and governments investing in transportation we need to introduce a transport agency which is responsible for managing the transport infrastructure. We assume that there is only one such an agency and that it can improve the existing transportation connection between the two regions. By doing so it reduces the commuting costs and influences residential location choices of the households, the rents and development decisions. Improvements in the transport infrastructure are assumed to be slower than building houses, but once in place they will last longer: it takes two periods to complete the investments and that the investment will last forever. The objective function of the agency is the sum of the welfare of the two regions, which is the sum of the total production (TP) minus the construction costs of housing (TC), the total commuting costs $\left(\mu\left(N_{BA}\right)^2\right)$ and the transport infrastructure costs $\left(INV\left(\mu\right)\right)$:

$$W\left(t\right)=W_A\left(t\right)+W_B\left(t\right)=TP_A\left(t\right)+TP_B\left(t\right)-TC_A\left(t\right)-TC_B\left(t\right)-\mu\left(N_{BA}\left(t\right)\right)^2-INV\left(\mu\right), \qquad (10)$$

where

$$TP_I\left(t\right)=\int_0^{n_I} MP_I\left(t\right)dn_I=\alpha_I n_I\left(t\right)-\frac{\beta_I}{2}\left(n_I\left(t\right)\right)^2,$$

$$TC_I\left(t\right)=\gamma b_I(t)+\frac{\delta}{2}(b_I(t))^2, \qquad I=A,B.$$

For the numerical simulation we will assume that the investment costs are inversely proportional to μ, higher investment levels corresponds to lower levels of μ.

2.5 Shock and equilibrium concepts

Now that all the agents have been defined we still need to specify the exogenous productivity shock and the assumptions on behavior and information for the different agents.

The shock is assumed to happen in region A in period $t = 1$ and is a permanent positive productivity shock, meaning that suddenly (e.g. due to exogenous investments in the zone) the marginal product function of region A will shift upwards from the start of period 1 onwards:

$$MP_A(t) = \alpha_A^S - \beta_A n_A\left(t\right), \quad \alpha_A^S > \alpha_A \quad t \geq 1. \tag{11}$$

In order to characterize the equilibria we need to specify the information and behavior of all agents that take decisions. For firms we assume that they always hire workers until the point where the marginal product equals the wage cost. Individuals have to take two decisions: where to live and where to work. As they are fully mobile between the two zones and as all individuals are identical, we need to satisfy the spatial equilibrium conditions given in e.g.(13): the location and work decisions will be an equilibrium when the utility of the individual can not be improved by moving to another location or other zone to work.

As long as transport agency are passive, it will be the behavior of the developers that will determine supply and equilibrium on the housing market and in the economy as a whole. We can construct different types of equilibriums: the perfect foresight, the delayed reaction and the myopic equilibria. Which equilibrium occurs depends on the information and anticipation rules of the developers. The equilibrium obtained when the shock is correctly anticipated ,both in timing and magnitude, and developers anticipate the future rental prices correctly is the perfect foresight equilibrium. If the shock is not anticipated but the developers have correct expectations on the future rental prices we end up in the delayed reaction equilibrium. Finally, when the shock is not anticipated and building decisions are based on past and current rents we have the myopic equilibrium. In Table 1 we summarize the three possibilities.

Table 1 Different equilibria

Perfect foresight	**Delayed reaction**	**Myopic equilibria**
Shock is fully anticipated	Shock is not anticipated but perfect foresight after the shock	Shock is not anticipated and decisions are based on weighted average of prices

In order to study the decisions on transport capacity we will, in addition, need to account for the anticipations of the transport agency on the buiding activity of the developers. It can either anticipate that the developers will react on a productivity shock or not. Moreover it can also anticipate or not that developers will adjust the housing stock to the change in transportation costs due to an investment.

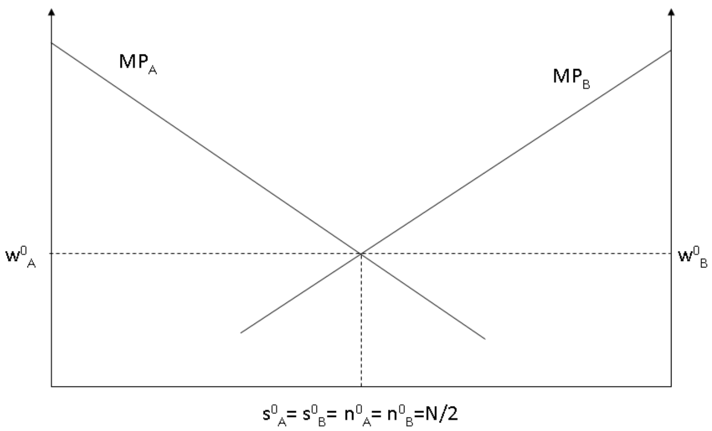
2.6 Initial equilibrium

Initially the two regions are identical (same amenities, productivity etc.). This means that, before the shock, exactly half of the total population will live in each region. As there is no difference in productivity between the two regions wages will be identical in both regions (see Figure 3) and both regions employ half of the population. As a result there will be no commuting and rents will be equal:

$$\bar{N}_A^0 = \bar{N}_B^0 = n_A^0 = n_B^0 = \frac{N}{2} \quad \text{and} \quad N_{BA}^0 = 0,$$

$$w_A^0 = w_B^0 = w^0 \quad , r_A^0 = r^0 \equiv r^0 \quad \text{and} \quad s_A^0 = s_B^0 = \frac{N}{2}.$$

Figure 3 Equilibrium in the absence of a shock and not yet accounting for housing and commuting costs



In the absence of a shock there is a housing stock that is the result of a steady state building activity:

$$s_i(t) = b_i(t-1) + b_i(t-2) = \frac{N}{2} \quad , i = A, B, \tag{12}$$

for example at the stationary state, the function $b(.)$ is constant and denoted by b^{st} and equal to $N/4$:

$$b_i(t-1) = b_i(t-2) = b^{st} \quad , i = A, B,$$

but it could follow a relation of the form

$$b_i(t-1) = \lambda \text{and} b_i(t-2) = \frac{N}{2} - \lambda \quad , i = A, B.$$

The magnitude of λ will be of no consequence for the rest of the analysis and will therefor be assumed to be $N/4$ which leads to a constant building activity.

3 Equilibrium without government investments in transport

We first assume that the government is completely passive and developers know that no investments will be made in the transportation network. As was already pointed out, the building decisions of the developers depend crucially on the future rent levels and how developers anticipate these levels. In the subsequent subsections we will assume different anticipation rules and analyze the resulting equilibria. First we assume that developers have perfect knowledge of the timing and magnitude of the shock (perfect foresight), next we assume that they do not anticipate the shock but once it occurs they anticipate the future rents correctly (delayed reaction). Finally we assume that developers are myopic, meaning that their prediction on future rent levels are based on actual and past rent levels. Households are assumed to react passively and to move without costs every period in function of their spatial equilibrium condition.

3.1 Perfect foresight equilibrium

3.1.1 Steady state equilibrium

We develop first the steady state equilibrium for the periods after the shock. Next we discuss the adaptation process. In a steady state after the shock, the rental prices, the stock and the population residing and working in each zone will remain constant over time. We proceed as follows. We first use the equilibrium conditions for the labour market e.g.(4)and the spatial equilibrium condition for the residents e.g.(13) to derive the demand for housing in each of the zones as a function of the rents in the two zones. Next we confront demand for housing units with the supply of housing units by the developers to determine the equilibrium number of residents as well as the number of commuters, the wages and the rental prices.

The demand for housing can be derived from the following spatial equilibrium condition where we have used the labour market equilibrium condition. We have in the steady state (for $t >> 1$):

$$\alpha^A - \beta\left[N_A(t) + N_{BA}(t)\right] - r_A(t) = \alpha - \beta\left[N_B(t)\right] - r_B(t) \qquad (13)$$

$$= \alpha^A - \beta\left[N_A(t) + N_{BA}(t)\right] - r_B(t) - \mu N_{BA}(t).$$

Where N_I is the number of people working and living in zone I. Inspecting the first and the third term implies that, in the presence of commuting we always have that the two rental prices are linked via the commuting costs ($\Delta r(t) = r_A(t) - r_B(t)$) :

$$\Delta r(t) = \mu N_{BA}(t). \tag{14}$$

The equality of the first and second term together with the constraint on population $(N = N_A(t) + N_B(t) + N_{BA}(t))$ implies

$$N_A(t) = \frac{(\alpha^A - \alpha) - \Delta r(t) + \beta N - 2\beta N_{BA}(t)}{2}.$$

Since we assume that there is only commuting form B to A, the number of people living and working in $A\,(N_A)$ will be equal to the number of residents in A. Using the relation between the two rental prices and the commuting costs gives us demand for housing units $N_A(t)$ as a function of the difference in rental prices:

$$N_A(t) = \frac{1}{2}\left[N + \frac{\alpha^A - \alpha}{\beta} - \frac{(2\beta + \mu)}{\beta\mu}\Delta r(t)\right]. \tag{15}$$

The increase in the number of residents of A will be larger when the productivity shock is larger because this increases the marginal product so that more workers are needed in zone A. Whenever the commuter costs (μ) are higher, it is more attractive to move to A rather than to commute so this also increases the population in A[2]. A higher rent in A compared to B will lead to a smaller increase of the population in zone A. Only the rent differences count and not the absolute rent because the number of individuals is fixed and all individuals need exactly one housing unit. The number of people working and living B is:

$$N_B(t) = N - N_A(t) - N_{BA}(t) = \frac{1}{2}\left[N - \frac{\alpha^A - \alpha}{\beta} + \frac{1}{\beta}\Delta r(t)\right].$$

For later use we also give the number of residents in A $(\bar{N}_A)$ and B $(\bar{N}_B)$:

---

[2] $$\frac{\partial N_A}{\partial \mu} = \frac{1}{\mu^2}(\Delta r(t)) > 0$$

$$\bar{N}_A\left(t\right)=N_A\left(t\right)=\frac{1}{2}\left[N+\frac{\alpha^A-\alpha^B}{\beta}-\left(\frac{2\beta+\mu}{\beta\mu}\right)\Delta r\left(t\right)\right], \tag{16}$$

$$\bar{N}_B\left(t\right)=N_B\left(t\right)+N_{BA}\left(t\right)=\frac{1}{2}\left[N-\frac{\alpha^A-\alpha}{\beta}+\frac{(2\beta+\mu)}{\beta\mu}\Delta r\left(t\right)\right]. \tag{17}$$

The difference between the two being:

$$\Delta\bar{N}\left(t\right)=\bar{N}_A\left(t\right)-\bar{N}_B\left(t\right)=\left[\frac{\alpha^A-\alpha}{\beta}-\frac{(2\beta+\mu)}{\beta\mu}\Delta r\left(t\right)\right].$$

The next step is to use the supply of housing as a function of the rental prices. In a steady state the stock of houses and rents remain constant over time:

$$r_A\left(t\right)=r_A\left(t+1\right),$$

$$b_A(t)=b_A(t+1)=\frac{N_A(t)}{2}.$$

Using e.g.(8) for the developer of housing for region A:

$$\rho r_A(t+1)+\rho^2 r_A(t+2)=\gamma+\delta\frac{\bar{N}_A(t)}{2},$$

and B:

$$\rho r_B(t+1)+\rho^2 r_B(t+2)=\gamma+\delta\frac{N_B(t)+N_{BA}\left(t\right)}{2}.$$

Combining the supply equations for zones A and B we can derive an expression for the difference in rental prices in equilibrium:

$$\Delta r^{SS}\left(t\right)=\frac{\delta}{\rho\left(1+\rho\right)}\frac{\bar{N}_A(t)-N_B(t)-N_{BA}(t)}{2}=\frac{\delta}{2\rho\left(1+\rho\right)}\left(2\bar{N}_A\left(t\right)-N\right). \tag{18}$$

Combining the supply condition with the demand condition e.g.(15), we find the equilibrium population in region A:

$$N_A^{SS}(t) = \bar{N}_A^{SS}(t) = \frac{N}{2} + \frac{1}{1+A/2}\frac{\left(\alpha^A - \alpha^B\right)}{2\beta}, \tag{19}$$

where,

$$\frac{1}{1+A/2} \equiv \frac{1}{1+\dfrac{\delta\left(\mu+2\beta\right)}{2\beta\mu\rho\left(1+\rho\right)}} = \frac{1}{1+\dfrac{2\beta+\mu}{\beta}B} \quad \text{with} \quad B \equiv \frac{\delta}{2\mu\rho\left(1+\rho\right)}.$$

We see that, in equilibrium, the number of residents in A is increasing in the productivity shock, is increasing in the commuting costs but decreasing in the slope of the building costs (δ). When the marginal cost of construction is constant, we see that the population of A is only determined by the productivity difference and that the discount factor and commuting costs play no role for the equilibrium. Substituting this back in e.g.(18) and using the relation between the rent difference and the number of commuters (e.g.(14)) we get

$$N_{BA}^{SS} = \frac{2B}{1+A/2}\frac{\left(\alpha^A - \alpha^B\right)}{2\beta}. \tag{20}$$

An increase in the the productivity shock will lead to more commuting since working in A is more attractive and, as could be expected, a decrease in commuting costs μ will increase the number of commuters.

3.1.2 The adaptation process

This is the steady state equilibrium after the shock. But we are also interested in the adaptation to the shock. Demand for housing and commuting reacts every period to the rental prices. The rental prices are, in the end determined by the construction activities so we need to study the behavior of developers. One of the important questions is to know when the housing market starts to adapt to the (known) shock: consists the dynamic equilibrium mechanism simply in building more houses in period 0 in zone A and less houses in zone B so that the housing market is again at steady state from period 1 onwards?

The developers decide on new houses in function of the expected rents. This means that all rental prices are linked. Using the developers equilibrium conditions e.g.(8), we have the following relations:

$$N_A(t) = s_A(t) = \frac{\rho}{\delta}\left[r_A(t-1) + (1+\rho)r_A(t) + \rho r_A(t+1)\right] - \frac{2}{\delta}\gamma,$$
$$N_A(t) - N_B(t) = \frac{\rho}{\delta}\left[\Delta r(t-1) + (1+\rho)\Delta r(t) + \rho\Delta r(t+1)\right]. \tag{21}$$

We see that all the periods are interlinked via the construction decisions that depend on the rental prices in the two coming periods.

In the perfect foresight equilibrium there will be no adaptation period, developers anticipate the shock correctly and will ensure that the steady state levels are satisfied when the shock takes place. The intuition goes as follow:

Take period 0. The anticipation of future rents implies that it pays for developers to increase the construction of houses in zone A in period 0 in order to satisfy the higher demand for houses in zone A in period 1 when the productivity shock increases the marginal product. Does this imply that the adaptation to the shock will consist for region A in the following oscillating construction activity:

1. higher building activity in period 0 so that in period 1, the new houses $b(0)$ + old houses $b(-1)$ equal the steady state stock s^{SS}?

2. lower building activity in period 1 as one only needs to replace old houses build in $t-1$

3. higher building activity in period 2 in order to replace the housing build in period 0

etc.

This would mean that the stock is fully adapted in period 1 and that this adaptation only takes one period.

Does it not make sense to smoothen the adaptation process by having a somewhat higher building activity already in period $t-1$? The answer is no for two reasons. First, the houses constructed in period t-1, will only serve one period after the shock at the beginning of period 1 while houses constructed in period 0 will serve in periods 1 and 2, two periods with higher demand in zone A. Second, a reason why developers would already increase construction in period -1 could be the construction cost function. The construction cost is increasing in b() and this tends to make smoothing more interesting. But because the marginal housing construction cost function is linear, there is no net gain by spreading the extra construction over several periods.

3.2 Non anticipated shock but perfect adaptation (delayed reaction)

In this case one observes an unanticipated shock in period 1. Commuting can react immediately but the change in the housing stock can only take effect in period 2. Future rental prices are perfectly anticipated. The new steady state is equal to the perfect foresight equilibrium. The only difference is that the adaptation is delayed by one period.

3.2.1 Myopic equilibrium

There are several versions of a myopic equilibrium. One commonly used assumption is that the rent in the next period is a weighted average of the rents in the past periods:

$$\tilde{r}_i(t+k) = \lambda r_i(t+k-1) + (1-\lambda) r_i(t+k-2).$$

This implies that developers now decide on construction on the basis of:

$$\left\{\rho \tilde{r}_i(t+1) + \rho^2 \tilde{r}_i(t+2)\right\} \geq \gamma + \delta(b_i(t)).$$

The optimal building strategy for the developer (e.g.(8)) then becomes

$$b_i(t) = \frac{\rho(1+\rho)}{\delta}\left[\lambda r_i(t) + (1-\lambda) r_i(t-1)\right] - \frac{\gamma}{\delta},$$

and the stock of housing in zone A at t is equal to

$$s_A(t) = \frac{\rho(1+\rho)}{\delta}\left[\lambda r_A(t-1) + r_A(t-2) + (1-\lambda) r_A(t-3)\right] - 2\frac{\gamma}{\delta}.$$

In the period of the shock there will be no adaptation yet, since $r_A(t<1) = r_B(t<1) \equiv r^0$ and ,

$$s_A(1) = s_B(1) = \frac{N}{2},$$

and housing stock will be equal in both regions. This means that there will be an over supply in region B and an under supply in A and some people will commute from B to A. The rent difference between the two regions can be derived by equating $\Delta s(t=1)$ and $\Delta\bar{N}(t=1)$:

$$\Delta s(1) = \Delta\bar{N}(1) \quad \Rightarrow \quad \Delta r(1) = \frac{\mu}{2\beta+\mu}\left(\alpha^A - \alpha^B\right) > 0$$

the rents in B will be lower to compensate the commuters and the higher wages in B (due to the shock). The number of commuters is then

$$N_{BA} = \frac{\left(\alpha^A - \alpha^B\right)}{2\beta+\mu}.$$

It can be shown that the rent difference will be greater than in the steady state meaning that there are more commuters. The reason is that the myopic developers did not to adjust the housing stock to accommodate the increase in demand in A. In the period just after the stock ($t = 2)$, the developers will react on the increased rents in region A by increasing construction in this region, indeed the difference in housing stock in the two regions at $t = 2$ is

$$\Delta s(2) = \lambda \frac{\rho(1+\rho)}{\delta} \Delta r(1) > 0.$$

We can derive the rent differences of all subsequent periods after equating the housing stock to the number of residents $\Delta s(t) = \Delta\bar{N}(t)$:

$$\Delta r(t) = \Delta r(1) - \frac{1}{A}\left[\lambda\Delta r(t-1) + \Delta r(t-2) + (1-\lambda)\Delta r(t-3)\right],$$

where

$$\Delta r(t) = 0 \quad \text{for}\, t < 1,$$

analytically it is not clear whether there is any convergence. It can, however, easily be checked that if the system convergence it will converge to the steady state Δr^{SS}. In the next section we illustrate that for reasonable values of the parameters of the model the system does convergence.

3.2.2 Numerical simulations

The values used for the numerical simulations are given in the next table:

Table 2 Variables values used in the simulations

Variables	**Value**
total population N	100
construction parameter γ	1
construction parameter δ	1
discount factor ρ	0.9
marginal production slope β	1
old productivity parameter in A (α_A)	100
new productivity parameter in A (α_A^S)	150
productivity parameter in B (α_B)	100
congestion parameter μ	0.5

In Table 3 and 4 the results are shown for $\lambda = 0.5$ and 1. The first column of each table represents the initial equilibrium, the shock takes place in period 1 (second column). The developers only react at this period and the stock of housing adapts from period 2 onwards. The last column is the steady state. (Note that we only give the first eight periods after the shock, there maybe need for more to really reach the steady state). As can be seen in Figure 4 where the rent differences for the two different lambda's are compared, the parameter λ does not seem crucial for the convergence. It seems that the value of the commuting costs plays a more important role. In Table 5 we give the simulation results for $\lambda = 0.5$ but this time $\mu = 0.1$ instead of 0.5 as previously and in Figure 5 the rent differences for the two different values of μ are plotted in a single graph.

Table 3 Simulations for myopic developers with μ=0.5 and λ=0.5

	SS1	SHOCK	adaptation								SS2
liv&workA	50.00	50.00	54.28	62.09	63.57	59.91	58.70	59.90	60.74	60.39	60.15
liv&workB	50.00	30.00	29.15	27.58	27.29	28.02	28.26	28.02	27.85	27.92	27.97
comm B to A	0.00	20.00	16.58	10.32	9.14	12.07	13.04	12.08	11.41	11.69	11.88
rentA	14.04	19.04	18.18	16.62	16.32	17.05	17.30	17.05	16.89	16.96	17.00
rentB	14.04	9.04	9.89	11.45	11.75	11.02	10.77	11.02	11.18	11.11	11.07
diff rent	0.00	10.00	8.29	5.16	4.57	6.04	6.52	6.04	5.70	5.84	5.94
Utility AA	35.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96
Utility BB	35.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96
Utility BA	35.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96
commuting costs	0.00	10.00	8.29	5.16	4.57	6.04	6.52	6.04	5.70	5.84	5.94
wage A	50.00	80.00	79.15	77.58	77.29	78.02	78.26	78.02	77.85	77.92	77.97
wage B	50.00	70.00	70.86	72.42	72.71	71.98	71.74	71.98	72.15	72.08	72.03
profit devA	364	364	434	578	608	536	513	536	552	545	540
profit dev B	364	364	300	199	183	226	241	226	215	220	223
profit firms A	1250	2450	2510	2622	2644	2591	2573	2591	2603	2598	2594
profit firms B	1250	450	425	380	372	392	399	393	388	390	391
welfare A	2163	3663	3900	4304	4376	4196	4134	4195	4237	4220	4208
welfare B	2163	3563	3372	2903	2797	3050	3126	3050	2996	3019	3034

Table 4 Simulations for myopic developers with μ=0.5 and λ=1

	SS1	SHOCK	adaptation								SS2
liv&workA	50.00	50.00	58.55	64.18	59.33	59.06	60.81	60.30	59.88	60.20	60.15
liv&workB	50.00	30.00	28.29	27.16	28.13	28.19	27.84	27.94	28.02	27.96	27.97
comm B to A	0.00	20.00	13.16	8.66	12.54	12.75	11.35	11.76	12.10	11.84	11.88
rentA	14.04	19.04	17.33	16.20	17.17	17.22	16.87	16.97	17.06	17.00	17.00
rentB	14.04	9.04	10.75	11.87	10.90	10.85	11.20	11.10	11.01	11.07	11.07
diff rent	0.00	10.00	6.58	4.33	6.27	6.38	5.68	5.88	6.05	5.92	5.94
Utility AA	35.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96
Utility BB	35.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96
Utility BA	35.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96
commuting costs	0.00	10.00	6.58	4.33	6.27	6.38	5.68	5.88	6.05	5.92	5.94
wage A	50.00	80.00	78.29	77.16	78.13	78.19	77.84	77.94	78.02	77.96	77.97
wage B	50.00	70.00	71.71	72.84	71.87	71.81	72.16	72.06	71.98	72.04	72.03
profit devA	364	364	510	620	525	520	553	543	535	541	540

	SS1	SHOCK	adaptation									SS2
profit dev B	364	364	243	176	233	236	215	221	226	222	223
profit firms A	1250	2450	2571	2652	2582	2579	2604	2596	2590	2595	2594
profit firms B	1250	450	400	369	396	397	387	390	393	391	391
welfare A	2163	3663	4126	4405	4166	4152	4241	4215	4194	4210	4208
welfare B	2163	3563	3135	2751	3087	3104	2991	3024	3052	3031	3034

Figure 4 The differences in rents during the adaptation period for λ=0.5 and λ=1

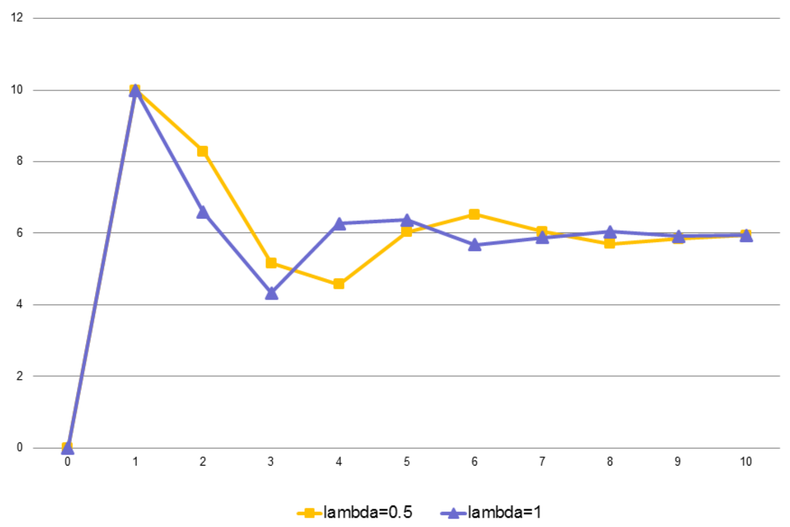
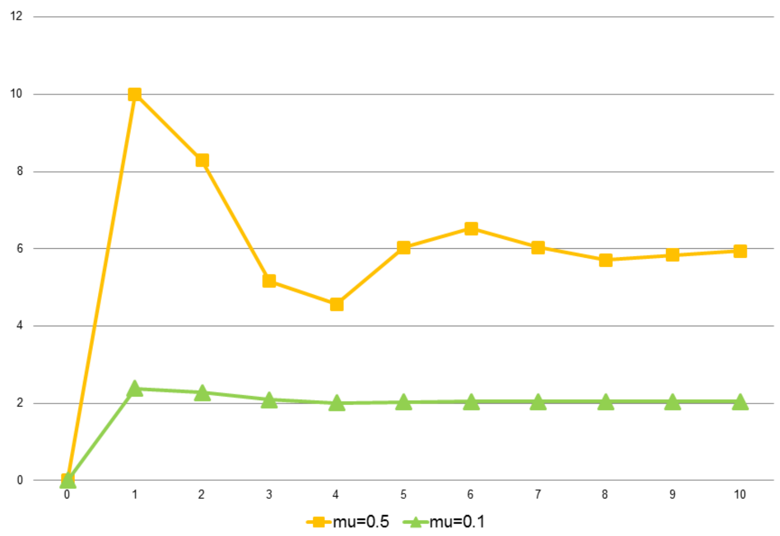


Table 5 Simulations results for myopic developers with μ=0.1 and λ=0.5

	SS1	SHOCK	adaptation									SS2
liv&workA	50.00	50.00	51.02	53.01	53.87	53.63	53.49	53.48	53.50	53.50	53.50
liv&workB	50.00	26.19	26.14	26.05	26.01	26.02	26.02	26.02	26.02	26.02	26.02
comm B to A	0.00	23.81	22.84	20.94	20.13	20.35	20.49	20.50	20.48	20.47	20.48
rentA	14.04	15.23	15.18	15.08	15.04	15.05	15.06	15.06	15.06	15.06	15.06
rentB	14.04	12.84	12.89	12.99	13.03	13.02	13.01	13.01	13.01	13.01	13.01
diff rent		2.38	2.28	2.09	2.01	2.04	2.05	2.05	2.05	2.05	2.05
Utility AA	35.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96
Utility BB	35.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96
Utility BA	35.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96	60.96
commuting costs	0.00	2.38	2.28	2.09	2.01	2.04	2.05	2.05	2.05	2.05	2.05
wage A	50.00	76.19	76.14	76.05	76.01	76.02	76.02	76.02	76.02	76.02	76.02
wage B	50.00	73.81	73.86	73.95	73.99	73.98	73.98	73.98	73.98	73.98	73.98

	SS1	SHOCK	adaptation								SS2
profit devA	364	364	380	413	427	423	421	421	421	421	421
profit dev B	364	364	348	319	306	310	312	312	312	311	311
profit firms A	1250	2724	2731	2738	2737	2736	2736	2736	2736	2736	2736
profit firms B	1250	343	340	338	339	339	339	339	339	339	339
welfare A	2163	3472	3595	3707	3688	3680	3683	3683	3683	3683	3683
welfare B	2163	3920	3801	3683	3703	3711	3709	3709	3709	3709	3709

Figure 5 The differences in rents during the adaptation period for λ=0.5 and λ=1



In conclusion, we found that the system does converge after more than eight periods. This has to be considered in relation to the lifetime of the building (2 periods) and the length of the rental period (1 period). If one period stands fot 10 years, and the housing is fully renewed after 20 years, the convergence after 80 years is a rather long period.

4 Optimum with transport investment

We now introduce a government transportation agency that can invest in transport infrastructure. To focus on the effect of including the housing market we first look into the case where the housing stock is constant and compare this to the case where developers are active players and react on a change in commuting costs. More specifically, we are interested in understanding how the inclusion of the housing market affects the optimal transport infrastructure investment rule. To compute the optimal investment level we assume that the government anticipates correctly the reaction of the developers. In the next section we will relax this assumption and analyze other equilibria.

4.1 Constant housing stock and rents

Assume that the government knows that due to a shock there will be a high demand for commuting. To avoid too high commuting costs it decides to improve the transportation network between the two regions. Once the shock occurs the investments are made and the commuting costs decrease[3]. In this section we will moreover assume that the stock of houses and the rents remain unchanged.

If rents can not adjust we cannot impose the spatial equilibrium conditions between residents of A and B (if we do we get that wages need to be equal and there will be no commuting), we can however impose that the utility of a commuter and someone working in B will be equal. This implies that

$$w_B = w_A - \mu N_{BA}.$$

Using e.g.(13) the number of commuters is:

$$N_{BA} = \frac{\left(\alpha^A - \alpha^B\right)}{2\beta + \mu}.$$

When commuting costs decrease, the number of commuters increases:

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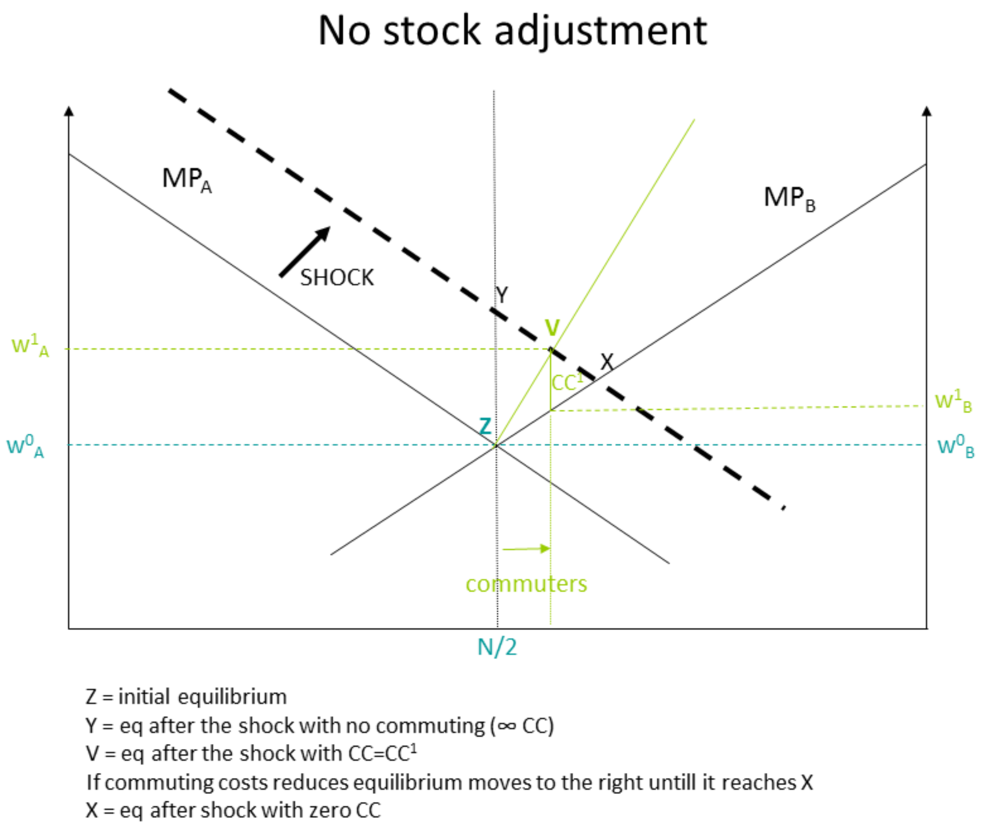
[3]So we assume that the government in charge of the investments has perfect knowledge on the timing and magnitude of the shock (perfect foresight).

$$\frac{\partial}{\partial \mu} N_{BA} = -\frac{N_{BA}}{(2\beta + \mu)} < 0.$$

As housing stock is constant an increase in commuters implies an increase in number of workers in region A which will lead to a decrease in wages in A (see Figure 6 where the wage in A decreases when going from point Y to V and X). This decrease implies that the utility of the residents of A will decrease. An increase in the wage in B compared to the case where no investments are made leads, on the other hand, to an increase of the utility of the residents of B (including the commuters)[4].

Figure 6 Equilibrium before and after a productivity shock in region A when rents and housing stock remain constant



[4] Note that there is still an increase in wages for both regions compared to the situation before the shock.

It is straightforward to derive the optimal investment rule. In the case that the housing stock remains constant we have

$$\frac{\partial \overline{N}_A}{\partial \mu} = \frac{\partial \overline{N}_B}{\partial \mu} = 0,$$

$$\frac{\partial TP_A}{\partial \mu} + \frac{\partial TP_B}{\partial \mu} = \left(\alpha_A - \alpha_B\right)\frac{\partial n_A}{\partial \mu} - \beta\left(n_A - n_B\right)\frac{\partial n_A}{\partial \mu},$$

$$\frac{\partial TC_A}{\partial \mu} + \frac{\partial TC_B}{\partial \mu} = 0.$$

Using the expression for the welfare given in e.g.(10) and optimizing w.r.t. $\mu,$ it turns out that the government will invest so that the marginal investment costs $\left(MINV\right)$ equals[5]:

$$-MINV^A = \left(\frac{2\beta}{2\beta + \mu}\right)\left(N_{BA}\right)^2 = \frac{2\beta\left(\alpha^A - \alpha^B\right)^2}{\left(2\beta + \mu\right)^3}. \tag{22}$$

The marginal benefits consists of two components, the marginal change in total production of the firms and the marginal change in total commuting costs. The marginal change in total commuting costs is squared in the number of commuters because the average cost is increasing in N_{BA}. The marginal change in total production is also a function of the square of the number of commuters. Since people can not move, the change in number of workers is only determined by the change in the number of commuters. For high values μ (a lot of congestion), there will be few commuters and a marginal change in the commuting costs will have less impact, the smaller the value of $\mu,$ the larger the impact. The marginal benefit is thus a decreasing function of $\mu.$ The larger the difference in productivity, the more people are willing to commute and the larger the benefits will be of an investment. The steeper the marginal product function of the firms (β larger), the less impact a change in commuting costs will have on the wages and on the total production and the smaller are the benefits of an investment. For later reference we will denote the resulting investment level as μ^A.

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[5]Note that since more investments in the capacity leads to a smaller $\mu,$ the marginal investment $\left(MINV\right)$ is negative.

4.2 Constant housing stock but rents can adjust

When rents can adjust the spatial equilibrium constraints for the individuals will hold and rents adjust until all individuals have equal utility. Compared to the case where the rents do not adjust, the residents of A will lose some utility since they see their rent increase, the opposite being true for the residents of region B. Note that this is the situation occurring at the period of the shock when developers are myopic; they did not anticipate the shock and thus housing stock is still at the old levels but rents can adjust.

4.3 Housing stock and rents can adjust

In the case where housing and rents can adjust freely we can make use of the steady state equations (13) We can show that in the steady state:

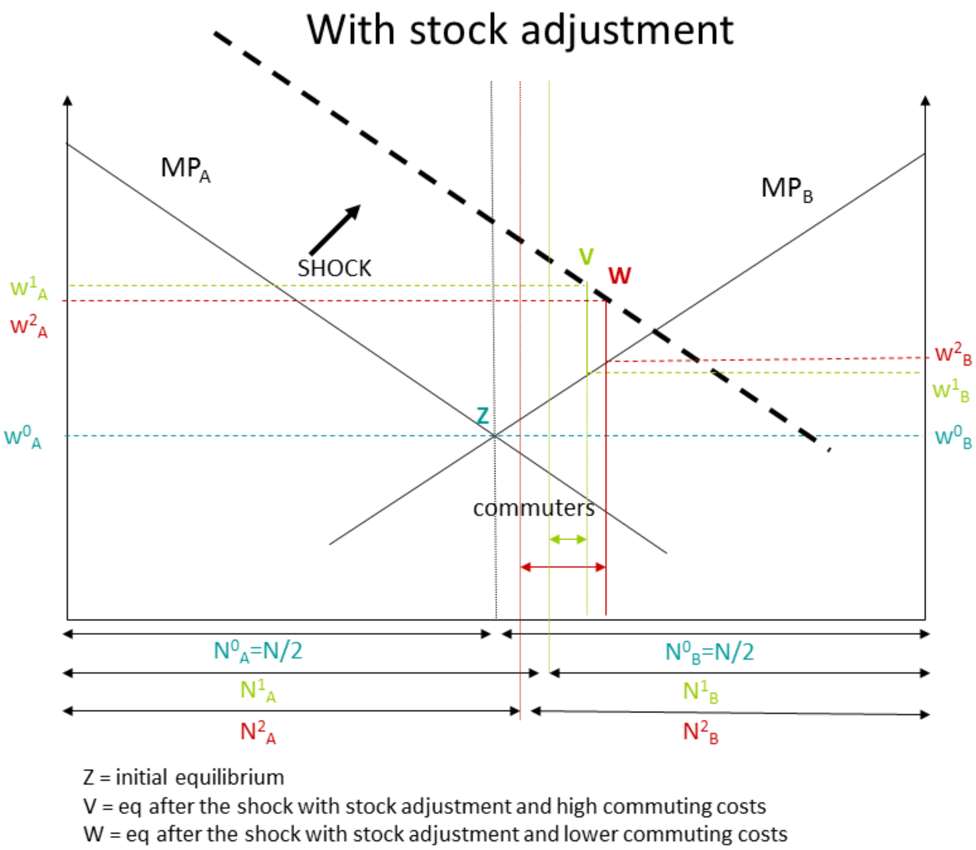
$$\frac{\partial N_{BA}^{SS}}{\partial \mu} < 0, \qquad \frac{\partial N_{A}^{SS}}{\partial \mu} > 0, \qquad \frac{\partial N_{B}^{SS}}{\partial \mu} > 0 \text{and} \frac{\partial \overline{N}_{B}^{SS}}{\partial \mu} < 0,$$

and

$$\left|\frac{\partial \overline{N}_{A}^{SS}}{\partial \mu}\right| < \left|\frac{\partial N_{BA}^{SS}}{\partial \mu}\right|.$$

A decrease in average commuting cost will make commuting more attractive and so, as one could expect, increases the number of commuters. This implies that region B becomes more attractive to live in and less people will choose to move to A. Region A will thus see its population decrease (on Figure 7 the population in A goes from N_A^1 to N_A^2). Although the total population in B increases there will be a decrease in the number of workers in B (due to the increase of commuters) and thus wages will increase (see Figure 7 where wages in B go from w_B^1 to w_B^2). In region A we see the opposite: the population decreases but due to the increase of the number of commuters, total workforce will increase and wages decrease.

Figure 7 Equilibrium before and after a productivity shock in region A when rents and housing stock can adjust.



Compared to the case where stock does not adjust the number of commuters will be inferior since some of the population has actually moved to A.

Next we look into the effects on the rents. Using

$$s_A = \frac{\rho 2(1+\rho)}{\delta} r_A - \frac{2}{\delta}\gamma = N_A \text{an } s_B = \frac{\rho 2(1+\rho)}{\delta} r_B - d\frac{2}{\delta}\gamma = N - N_A,$$

we can derive

$$\frac{\partial r_A}{\partial \mu} = \frac{\delta}{2\rho(1+\rho)} \frac{\partial N_A^{SS}}{\partial \mu} = -\frac{\partial r_B}{\partial \mu},$$

rents in A decrease (to off-set the decrease in wages), while rents in B increase by just the same amount. Note that the total utility of all residents remains unchanged when investments are made since rents will adjust to annihilate the effect of the changes in wages and commuting costs.

The optimal investment rule will now look quite differently. The number of commuters is in this case equal to N_{BA}^{SS}, deriving this w.r.t. μ yields:

$$\frac{\partial N_{BA}^{SS}}{\partial \mu} = -\left[\frac{\beta + \mu B}{\mu \beta}\right] \frac{1}{1 + A/2} N_{BA}^{SS} < 0.$$

The derivatives of the total production and construction costs are now:

$$\frac{\partial TP}{\partial \mu} = \left[\left(\alpha^A - \alpha^B\right) - \beta\left(n_A - n_B\right)\right]\left(\frac{\partial \bar{N}_A}{\partial \mu} + \frac{\partial N_{BA}}{\partial \mu}\right),$$

and

$$\frac{\partial TC}{\partial \mu} = \frac{\delta}{4}\left(N_A - N_B\right)\frac{\partial N_A}{\partial \mu}.$$

Putting this together, we get as optimal investment rule:

$$-MINV^B = \Delta\alpha\left[\frac{\partial \bar{N}_A}{\partial \mu} + \frac{\partial N_{BA}}{\partial \mu}\right] - \beta\left(N_A + 2N_{BA} - N_B\right)\left[\frac{\partial N_A}{\partial \mu} + \frac{\partial N_{BA}}{\partial \mu}\right],$$

$$+\frac{\delta}{4}\left(N_A - N_B\right)\frac{\partial N_A}{\partial \mu} - \left(N_{BA}\right)^2 - 2\mu N_{BA}\frac{\partial N_{BA}}{\partial \mu}.$$

Again, we can express everything in terms of $\left(N_{BA}\right)^2$:

$$-MINV^B = \left[2B - 1 - \frac{\delta}{4\mu B}\right]\left(\frac{1}{1 + A/2}\right)\left(N_{BA}^{SS}\right)^2. \tag{23}$$

It can be shown that the right-hand side of eq(23) is always smaller than the right-hand side of eq(22): since there will be less commuters when the housing stock can adjust, the marginal benefits of a reduction of the commuting costs will be smaller, the optimal investment level $\left(\mu^B\right)$ will therefore be smaller than in the case where the stock does not adjust. Governments who do not take the developers reaction into account are thus likely to overinvest.

5 The developers, the tranport agency, their anticipations and the resulting equilibrium

In the previous section we have studied the optimal transport investment when the transport agency anticipated correctly the reaction of developpers on the productivity shock and on the transport investment. In this section we study the role of expectations of the transport agency and the developers for the housing and transport infrastructure decisions. We do thisfor the steady state.

Consider first the developers. They realize that the willingness to pay for additional housing in region A depends on the transport infrastructure in place. As the transport infrastructure has a longer life than housing, we know that, in the steady state, the housing stock will depend on the transport infrastructure stock in place. As long as we are only interested in the steady state we can therefore concentrate on the anticipations and decisions of the transport agency. This will dictate the approach in this section.

Incorrect anticipation by the transport agency can be based on over- or underestimate of the adjustment of the housing stock adjustment to a shock. As the transport agency has to make only one decision on the size of the infrastructure we need to specify what information is used by the transport agency. In the previous section we already studied two extreme cases for the transport infrastructure investment. The first was the case where the housing stock does not adjust. The second case had an optimal adjustment of the infrastructure stock. The transport agency could also anticipate (wrongly) that the developers react to the productivity shock but do not anticipate the (slower) adjustment of the transport infrastructure. In this case the transport agency assumes that the developers do anticipate the shock and will built new houses in A up to the steady state level $s_A^{SS}(\mu)$ equal to eq(19), but since the agency does not expect the developers to anticipate an investment in the transport capacity it will assume that μ is equal to the intial capacity μ^0. To find the level of investment chosen by the agency we maximize welfare for a given stock of houses equal to $\hat{s}_A^{SS}$ which is assumed to be fixed (not dependend on μ).

For a given stock s_A the government chooses μ that maximizes the welfare function. Since the huosing stock is assumed to be constant we have again that

$$\frac{\partial \overline{N}_A}{\partial \mu} = \frac{\partial \overline{N}_B}{\partial \mu} = 0,$$

$$\frac{\partial TP_A}{\partial \mu} + \frac{\partial TP_B}{\partial \mu} = (\alpha_A - \alpha_B)\frac{\partial N_{BA}}{\partial \mu} - \beta(n_A - n_B)\frac{\partial N_{BA}}{\partial \mu},$$

$$\frac{\partial TC_A}{\partial \mu} + \frac{\partial TC_B}{\partial \mu} = 0.$$

The number of commuters is expected to be equal to eq(20)

$$N_{BA} = \frac{\beta}{2\beta + \mu}\left[\frac{N}{2} + \frac{\Delta\alpha}{2\beta} - \hat{s}_A\right],$$

where the stock of housing in region A is expected to be:

$$\hat{s}_A = \frac{N}{2} + \frac{2\beta\mu^0\rho(1+\rho)}{2\beta\mu^0\rho(1+\rho) + \delta\left(\mu^0 + 2\beta\right)}\frac{\Delta\alpha}{2\beta}.$$

Subtituting this in the f.o.c. we get:

$$-MINV^C = \frac{4\beta + \mu}{2\beta + \mu}\left(N_{BA}^2\right)^2. \tag{24}$$

We can now sum up the three cases we discussed where the transport infrastructure is based on the following three assumptions on the anticipations of the transport agency :

- no housing adjustment to shock and transport infrastructure $\left(\mu^A\right)$
- housing adjusts to the shock but not to the changes in transport infrastructure $\left(\mu^C\right)$
- correct anticipations of the developers reactions to the shock and the infrastructure extension $\left(\mu^B\right)$

where $\mu^A < \mu^B < \mu^C$. The last inequality is not easy to proove but it is possible to show that $N_{BA}^C < N_{BA}^B < N_{BA}^A$. The (expected) marginal benefit of a capacity extention will therefor be less in the case where the transport agency does not take into account that developers will react on an extra investment than when it does and will invest less which results in a higher value of μ. We summarize the results in the Table 6 :

Table 6 Summary of the results according to the anticipations of the transport agency

Description	**Anticipation of the transport agency on the housing stock change**	**Anticipation of the transport agency on the number of commuters**	**Resulting investment levels**
no housing adjustment	$\Delta \overline{ss}_A^A = 0$	$\overline{N}_{BA}^A$	μ^A is large
	$\wedge$	$\vee$	$\wedge$
optimal adjustment to shock and investment	$\Delta \overline{ss}_A^B = 0$	$\overline{N}_{BA}^B$	μ^B is intermediate
	$\wedge$	$\vee$	$\wedge$
adjustment to housing stock but not to transport investment	$\Delta \overline{ss}_A^C = 0$	$\overline{N}_{BA}^C$	μ^C is intermediate

6 Conclusion

In this paper we investigate the role of anticipations in the housing market and the possible interplay with transport investments. We use a simple two region model where one region model where one region is subject to an unanticipated productivity shock that increases the demand for labour in that region. The demand for labour can be met by additional commuting, by more housing or by additional investment in transport. When no additional investment in transport is possible, the result depends on the expectations of the developers. Developers with myopic expectations arrive also at the optimal housing stock but need more cycles to reach the optimum.

When we add a transport agency that can invest to improve the transport infrastructure between the two regions there is a coordination problem. Transport investment tend to have longer construction periods and lifetimes than housing investments. In this case the steady state equilibrium will mainly depend on the anticipation of the transport agency. Depending on the anticipations, we can see over or underinvestments in transport infrastructure.

The model is a simplistic two region model with only one mode of transport. Interesting extensions include the consideration of the introduction of regional governments that can tax income and commuting flows, decide on the building permits etc. See also de Palma and Fosgerau (2011) on this issue.

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Appendix

A summary of the notation used in the paper can be found in the following table

Table 7 Notation

Symbol	Description
N_I	residents of region I working in region I
N_{BA}	commuters: residents of region B working in region A
$\bar{N}_I$	residents of region I working in either region I or J
N	total popunlation (fixed)
n_I	workers in region I
μ	slope of timecost function (inverse of capacity)
w_I	wage received in region I
α_I	intercept of marginal product function of firms in I
β_I	slope of marginal product function of firms in I
$s_I(t)$	stock of houses in region I at time t
$b_I(t)$	number of houses built in region I in period t
$r_I(t)$	rent of houses in region I at time t
ρ	discount factor
γ	intercept of marginal cost function of the developers in region I
δ	slope of marginal cost function of the developers in region I

$U_I(w_I, r_I, \mu)$	Utiltiy of residents of I
$MP_I(n_I)$	Marginal product of firms in I
$TP_I(n_I)$	Total product function of firms in I
$TC_I(b_I)$	Total cost function of building houses in I
Π_I	Profit of developers in I
W	Welfare function
$INV(\mu)$	Cost of investments in transportation