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Optimal housing consumption and portfolio choice with exogenous random shocks

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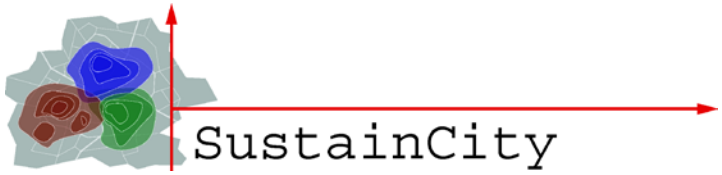

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Abstract

We analyze the portfolio choice and the housing investment of an investor over his life span. The individual can decide how much to invest on financial assets (bonds and stocks), on housing units owned as well as perishable goods. This paper examines the impact of new and major sources of risk. Such major events correspond for example to a sudden loss of employment or for events such as divorce. We assume CES utility function for the consumption of housing and perishable goods and describe risk aversion with CRRA specification. The final condition is driven by a bequest left at the end of the life-cycle. We first start with a financial portfolio optimization problem and show how the standard solution is qualitatively and quantitatively affected by independent and exogenous random shocks. Then, we set up a continuous time-model including real estate investment opportunities. For the CRRA utility functions, we compute explicitly the optimal solutions. Then, we compute the compensating variations between various optimization frameworks such as the total ignorance of the random shock (myopic behavior), the perfect information about the shock's impact (perfect foresight) and finally the knowledge of the probability distribution of the shock (rational expectation). This allows to en-light the main differences between these three basic cases and to provide the monetary utility losses due to partial information about shock randomness.

Keywords

Portfolio optimization; Housing consumption; Real estate; S hock randomness; Compensating variations

Preferred citation style

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Executive Summary

Standard portfolio and consumption optimization models do not sufficiently take account of potential additional shock that can occur suddenly and imply significant changes in optimal strategies. Such shock can correspond for instance to loss of employment, divorce or death of one member of the couple. In this paper, we introduce this special source of risk corresponding to exogenous random shocks, which are independent from the financial and real estate assets. We show how standard optimization solutions are modified by independent and exogenous random shocks.

We analyze the portfolio choice and the housing investment of an investor over his life span. The individual can decide how much to invest on financial assets (bonds and stocks), on housing units owned as well as perishable goods. We assume CES utility function for the consumption of housing and perishable goods and describe risk aversion with CRRA specification. The final condition is driven by a bequest left at the end of the life-cycle. We have started with a financial portfolio optimization problem. We illustrate how the standard solution is modified by independent and exogenous random shocks. We consider three basic cases: the first one corresponds to an individual who ignore totally the random shocks: he optimizes his investments as if no future shock can happen (myopic behavior); the second one considers that the individual is initially perfectly informed about the future consequences of the shock (perfect foresight); finally, we assume that the individual knows that a shock may occur but has only a probability distribution of the shock (rational expectation).

For the CRRA utility functions, we compute the explicit optimal solutions. To emphasize the main differences between these three basic cases, we introduce a quantitative measure based on the standard economic concept of "compensating variation" to measure the monetary losses for various optimization frameworks such as the total ignorance of the random shock, the perfect information about the shock's impact and finally the knowledge of the probability distribution of the shock. This allows to en-light the main differences between these three basic cases and to provide the monetary utility losses due to partial information about shock randomness

Then, we set up a continuous time-model including real estate investment opportunities. This model allows the explicit computation of closed-form solutions for the optimal life-cycle portfolio, housing and consumption strategies for the three basic cases. In the CES and CRRA case, we show how these optimal strategies are modified for the three cases. We prepare the exact solutions for each case. In particular, we determine exactly how the potential shock changes or not the optimal strategies. We prove that the financial shares are not affected while the consumption ratios are reduced when the individual has a significant relative risk aversion.

1 Introduction

The standard literature about dynamic consumption and portfolio management is based on seminal papers of Samuelson (1969) and Merton (1969, 1971). The common assumption to model investor choices is the expected utility criterion, which has been widely developed in the financial portfolio theory. Most of theoretical results have been determined in the continuous-time setting. Karatzas *et al.* (1986) and Cox and Huang (1989) have determined the optimal strategies of an investor maximizing the expected utility of his consumption and portfolio value, when assets prices are assumed to be diffusion processes. Explicit analytical results can be provided in this framework for standard cases as proved by Merton (1969). Cvitanic and Karatzas (1996) have solved this problem with constraints on portfolio. Such results have been further extended for instance by taking account of market incompleteness, labor income, stochastic horizon...(for a survey, see e.g. Karatzas and Shreve, 1998; Campbell and Viceira, 2002; Prigent, 2007).[1] Optimal housing consumption has been further investigated, for example by Cocco (2005) and Cocco *et al.* (2005). The optimal housing investment varies generally much more than the purely financial investments in bonds and stocks. As illustrated by Kraft and Munk (2011), house prices and labor incomes are highly correlated and labor income risk can vary with age. Using life-cycle patterns in expected income growth estimated for distinct educational groups by Cocco *et al.* (2005), Kraft and Munk (2011) show for example that typically college graduates "should less invest in the housing asset early in life and enter into owner-occupied housing later in life than typical less-educated individuals". As in Yao and Zhang (2005), they show that significant welfare gains are obtained by allowing renting and that the flexibility between renting and owning changes the optimal investment strategy.

In this paper, we introduce another source of risk corresponding to exogenous random shocks, which are independent from the financial and real estate assets. They can correspond to a sudden loss of employment or other events such as divorce.

In Section 2, we first deal with a purely financial portfolio optimization problem. We illustrate how the standard solution is modified by independent and exogenous random shocks. We consider three basic cases: the first one corresponds to an individual who ignore totally the random shocks: he optimizes his investments as if no future shock can happen (myopic behavior); the second one considers that the individual is initially perfectly informed about the future consequences of the shock (perfect foresight); finally, we assume that the individual knows that a shock may occur but has only a probability distribution of the shock (rational expectation). For the CRRA utility functions, we determine the explicit optimal solutions. To emphasize the main differences between these three basic cases, we introduce a quantitative measure based on the standard economic concept of "compensating variation" to measure the monetary losses

[1] For implementations of methods to compute numerically intertemporal asset allocations, we refer to Detemple, Garcia and Rindisbacher (2003, 2005).

due to total ignorance or partial information about shock randomness.[2]

In Section 3, we set up a continuous-time model including real estate investment opportunities, as described in Kraft and Munk (2011).

In Section 4, this model allows the explicit computation of closed-form solutions for the optimal life-cycle portfolio, housing and consumption strategies for the three basic cases. Section 5 summarizes and concludes.

2 Basic example of a model with an exogenous shock

In what follows, we consider a portfolio investment model with two basic financial assets: one riskless denoted by B (the "Bond") and one risky denoted by S (The "Stock"). A random shock can occur at maturity and reduce the terminal wealth V_T.

2.1 Asset dynamics and individual's strategies

We assume that the two basic financial assets follows the dynamics respectively given by:

$$\begin{cases} \frac{dB_t}{B_t} = rdt, \\ \frac{dS_t}{S_t} = \mu dt + \sigma dW_t, \end{cases}$$

where r denotes the riskless interest rate, μ is the instantaneous rate of the risky return and σ denotes the volatility. The process W is assumed to be a standard Brownian motion.

The individual's investment and consumption strategies are defined by the asset shares: $(w_t^B = 1 - w_t^S)_t$ and $(w_t^S)_t$ and the consumption rate $(c_t)_t$.

The portfolio value dynamics are given by:

$$dV_t = V_t \left(rdt + w_t^S \left[(\mu - r)\, dt + \sigma dW_t \right] \right) - c_t dt.$$

The available information is generated by the Brownian motion (filtration $(\mathcal{F}_t)_t$).

Therefore, the financial market is complete and there exists only one risk-neutral probability $\mathbb{Q}$, characterized by its Radon-Nikodym derivative $(\eta_t)_t$ given by:

$$\eta_t = \mathbb{E}[\frac{d\mathbb{Q}}{d\mathbb{P}}/\mathcal{F}_t] = \exp[-\left(\frac{\mu - r}{\sigma}\right) W_t - \frac{1}{2}\left(\frac{\mu - r}{\sigma}\right)^2 t]$$

The utility of portfolio value is defined by:

$$\mathbb{E}\left[\int_0^T u(c_t)dt + U(V_T)\right]$$

[2] See de Palma and Prigent (2008, 2009) for the notion of compensating variation and its illustration in portfolio management.

with the usual CRRA assumptions: $u(c) = e^{-\rho t}\frac{c^{1-\gamma}}{1-\gamma}$ and $U(V) = \frac{V^{1-\gamma}}{1-\gamma}$ with $\gamma \neq 1$.

We assume that the investor may suffer from losses due to an exogenous factor at maturity. At time T, the investor's wealth may decrease due to an exogenous random shock. This is modelled by a random variable X_T. The random variable X_T is assumed to take its values in $[0, 1]$. For example, the probability distributions of X_T can be of Bernoulli type :

$$\mathbb{P}[X_T = 1] = 1 - p \text{ and } \mathbb{P}[X_T = a] = p, \text{ with } 0 < a < 1.$$

2.2 Myopic behavior

The investor behaves as if a shock could not occur. Thus, his objective is given by:

$$Max_{C,w^S}\ \mathbb{E}\left[\int_0^T e^{-\rho s}u(C_s)ds + U(V_T)\right].$$

Denote $j = (u')^{-1}$ and $J = (U')^{-1}$.
The optimal strategies are given by:

1. Optimal consumption:

$$C^*(t) = j\left[\lambda\eta(t)e^{(\rho-r)t}\right].$$

2. Optimal portfolio value:

$$V_T^* = J\left[\lambda\eta_T e^{-rT}\right]$$

The Lagrange parameter λ corresponding to the budget constraint satisfies $\lambda = f^{-1}(V_0)$, where the function f is defined by:

$$f(y) = \mathbb{E}_P\left[\int_0^T j\left(y\eta_t e^{(\rho-r)t}\right)\eta_t e^{-rt}dt + J\left(y\eta_T e^{-rT}\right)\eta_T e^{-rT}\right].$$

Assume for example that $U(x) = u(x) = x^{1-\gamma}/(1-\gamma)$ with $\gamma \neq 1$. Then, we get:

$$\lambda^{-\frac{1}{\gamma}} = \frac{V_0}{\mathbb{E}_P\left[\int_0^T \left(\eta_t e^{(\rho-r)t}\right)^{-\frac{1}{\gamma}}\eta_t e^{-rt}dt + \left(\eta_T e^{-rT}\right)^{-\frac{1}{\gamma}}\eta_T e^{-rT}\right]},$$

with:

$$\mathbb{E}_P\left[\int_0^T \left(\eta_t e^{(\rho-r)t}\right)^{-\frac{1}{\gamma}}\eta_t e^{-rt}dt + \left(\eta_T e^{-rT}\right)^{-\frac{1}{\gamma}}\eta_T e^{-rT}\right] =$$

$$\frac{\exp\left[\left(A-\left(\frac{\rho-r}{\gamma}\right)-r\right)T\right]-1}{\left[\left(A-\left(\frac{\rho-r}{\gamma}\right)-r\right)T\right]}+\exp\left[\left(A-r\left(\frac{\gamma-1}{\gamma}\right)\right)T\right],$$

and

$$A=-\frac{1}{2}\left(\frac{\gamma-1}{\gamma^2}\right)\left(\frac{\mu-r}{\sigma}\right)^2.$$

3.

$$w_S^*=\frac{1}{\gamma}\left(\frac{\mu-r}{\sigma^2}\right).$$

4. Introduce the functions

$$\Phi\left(\rho,z;r,\mu,\sigma,T,\gamma\right)=$$

$$\left(\frac{\exp\left[\left(A-\left(\frac{\rho-r}{\gamma}\right)-r\right)T\right]-1}{\left[\left(A-\left(\frac{\rho-r}{\gamma}\right)-r\right)T\right]}+z\exp\left[\left(A-r\left(\frac{\gamma-1}{\gamma}\right)\right)T\right]\right)$$

and

$$\Psi\left(\rho,z;r,\mu,\sigma,T,\gamma\right)=$$

$$\left(\frac{\exp\left[\left(A-\left(\frac{\rho-r}{\gamma}\right)-r\right)T\right]-1}{\left[\left(A-\left(\frac{\rho-r}{\gamma}\right)-r\right)T\right]}+z\exp\left[\left(A-r\left(\frac{\gamma-1}{\gamma}\right)\right)T\right]\right)^{\gamma}$$

The utility of the optimal strategy is given by:

$$\mathbb{E}\left[\int_0^T e^{-\rho s}u(j\left[\lambda\eta_s e^{(\rho-r)s}\right])ds+U\left(J\left[\lambda\eta_T e^{-rT}\right]\right)\right]=$$

$$\frac{\lambda^{\frac{\gamma-1}{\gamma}}}{(1-\gamma)}\mathbb{E}\left[\int_0^T e^{-\rho s}\left[\eta_s e^{(\rho-r)s}\right]^{\left(\frac{\gamma-1}{\gamma}\right)}ds+\left(\eta_T e^{-rT}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}\right]=$$

$$\frac{\lambda^{\frac{\gamma-1}{\gamma}}}{(1-\gamma)}\left[\frac{\exp\left[\left(A+(\rho-r)\left(\frac{\gamma-1}{\gamma}\right)-\rho\right)T\right]-1}{\left[\left(A+(\rho-r)\left(\frac{\gamma-1}{\gamma}\right)-\rho\right)T\right]}+\exp\left[\left(A-r\left(\frac{\gamma-1}{\gamma}\right)\right)T\right]\right]$$

with

$$\lambda^{\frac{\gamma-1}{\gamma}}=\frac{\left(V_0\right)^{1-\gamma}}{\left(\frac{\exp\left[\left(A-\left(\frac{\rho-r}{\gamma}\right)-r\right)T\right]-1}{\left[\left(A-\left(\frac{\rho-r}{\gamma}\right)-r\right)T\right]}+\exp\left[\left(A-r\left(\frac{\gamma-1}{\gamma}\right)\right)T\right]\right)^{1-\gamma}}.$$

Finally, the utility of the optimal strategy is equal to:

$$\mathcal{U}=\frac{V_0^{1-\gamma}}{(1-\gamma)}\Phi\left(\rho,1;r,\mu,\sigma,T,\gamma\right)^{\gamma}.$$

5. The utility of the optimal strategy taking account of the possible shock is given by:

$$\mathbb{E}\left[\int_0^T e^{-\rho s}u(j\left[\lambda\eta_s e^{(\rho-r)s}\right])ds + U\left(J\left[\lambda\eta_T e^{-rT}\right]X_T\right)\right] =$$

$$\frac{\lambda^{\frac{\gamma-1}{\gamma}}}{(1-\gamma)}\mathbb{E}\left[\int_0^T e^{-\rho s}\left[\eta_s e^{(\rho-r)s}\right]^{\left(\frac{\gamma-1}{\gamma}\right)}ds + \left(\eta_T e^{-rT}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}\mathbb{E}\left[X_T^{1-\gamma}\right]\right] =$$

$$\mathcal{U}^M = \frac{V_0^{1-\gamma}}{(1-\gamma)}\frac{\Phi\left(\rho, \mathbb{E}\left[X_T^{1-\gamma}\right]; r, \mu, \sigma, T, \gamma\right)}{\Phi\left(\rho, 1; r, \mu, \sigma, T, \gamma\right)^{1-\gamma}}.$$

2.3 Perfect foresight

The individual knows at the beginning of the period if the shock will occur or not and consumes/invests accordingly. If the investor knows that the shock X_T is equal to a given value x in $[0,1]$, then his objective is

$$Max_{C,w^S}\ \mathbb{E}\left[\int_0^T e^{-\rho s}u(C_s)ds + U\left(xV_T\right)\right].$$

The optimal strategies are given by:

1. Optimal consumption:

$$C^*(t) = j\left[\lambda(x)\eta(t)e^{(\rho-r)t}\right].$$

2. Optimal portfolio value:

$$V_T^* = \frac{1}{x}J\left[\frac{\lambda(x)}{x}\eta_T e^{-rT}\right]$$

The Lagrange parameter $\lambda(x)$ corresponding to the budget constraint satisfies $\lambda(x) = f^{-1}(x, V_0)$, where the function f is defined by:

$$f(x,y) = \mathbb{E}_P\left[\int_0^T j\left(y\eta_t e^{(\rho-r)t}\right)\eta_t e^{-rt}dt + \frac{1}{x}J\left(\frac{y}{x}\eta_T e^{-rT}\right)\eta_T e^{-rT}\right].$$

Assume for example that $U(x) = u(x) = x^{1-\gamma}/(1-\gamma)$ with $\gamma \neq 1$. Then, we get:

$$\lambda(x)^{-\frac{1}{\gamma}} = \frac{V_0}{\left(\frac{\exp\left[\left(A-\left(\frac{\rho-r}{\gamma}\right)-r\right)T\right]-1}{\left[\left(A-\left(\frac{\rho-r}{\gamma}\right)-r\right)T\right]} + \left(\frac{1}{x}\right)^{\frac{\gamma-1}{\gamma}}\exp\left[\left(A - r\left(\frac{\gamma-1}{\gamma}\right)\right)T\right]\right)},$$

3. The optimal weight invested on the risky asset satisfies:

$$w_S^* = \frac{1}{\gamma}\left(\frac{\mu - r}{\sigma^2}\right).$$

4. The utility of the optimal strategy knowing that the shock is equal to x is given by:

$$\mathbb{E}\left[\int_0^T e^{-\rho s} u(j\left[\lambda(x)\eta_s e^{(\rho-r)s}\right])ds + U\left(J\left[\lambda(x)\frac{1}{x}\eta_T e^{-rT}\right]\right)\right] =$$

$$\frac{\lambda(x)^{\frac{\gamma-1}{\gamma}}}{(1-\gamma)}\mathbb{E}\left[\int_0^T e^{-\rho s}\left[\eta_s e^{(\rho-r)s}\right]^{\left(\frac{\gamma-1}{\gamma}\right)} ds + \left(\frac{1}{x}\eta_T e^{-rT}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}\right].$$

Finally, the utility of the optimal strategy is equal to:

$$\mathcal{U}^{Pi}(x) = \frac{V_0^{1-\gamma}}{(1-\gamma)}\Psi\left(\rho, (1/x)^{\left(\frac{\gamma-1}{\gamma}\right)}; r, \mu, \sigma, T, \gamma\right).$$

2.4 Optimal behavior with rational expectation

The individual knows at the beginning of the period that a shock can occur but he does not know exactly the shock value. He has only a probability distribution about the random values of the shock. He consumes/invests accordingly. If the investor knows that the shock X_T is equal to a given value x in $[0, 1]$, then his objective is

$$Max_{C,w^S}\ \mathbb{E}\left[\int_0^T e^{-\rho s} u(C_s)ds + U\left(X_T V_T\right)\right].$$

The optimal strategies are given by:

1. Optimal consumption:

$$C^*(t) = j\left[\lambda_X \eta(t) e^{(\rho-r)t}\right].$$

2. Optimal portfolio value:

$$V_T^* = J_X\left[\lambda_X \eta_T e^{-rT}\right],$$

where the function J_X is the inverse of the function U_X defined by:

$$U_X(v) = \mathbb{E}\left[U\left(X_T\ v\right)\right] = \int U\left(x\ v\right)\mathbb{P}_{X_T}(dx),$$

where $\mathbb{P}_{X_T}(.)$ is the probability distribution of the random variable X_T. Since U is strictly increasing, U_X is also strictly increasing and invertible.

3. The Lagrange parameter λ_X corresponding to the budget constraint satisfies $\lambda_X = f_X^{-1}(V_0)$, where the function f_X is defined by:

$$f_X(y) = \mathbb{E}_{\mathbb{P}}\left[\int_0^T j\left(y\eta_t e^{(\rho-r)t}\right)\eta_t e^{-rt}dt + J_X\left(y\eta_T e^{-rT}\right)\eta_T e^{-rT}\right].$$

Assume for example that $U(x) = u(x) = x^{1-\gamma}/(1-\gamma)$ with $\gamma \neq 1$. Then, we get:

$$U_X(v) = \frac{1}{(1-\gamma)}\mathbb{E}\left[U\left(X_T\ v\right)\right] = \frac{1}{(1-\gamma)}v^{1-\gamma}\mathbb{E}\left[X_T^{1-\gamma}\right],$$

$$U_X'(v) = v^{-\gamma}\mathbb{E}\left[X_T^{1-\gamma}\right],$$

$$V_T^* = J\left[\frac{\lambda_X\eta_T e^{-rT}}{\mathbb{E}\left[X_T^{1-\gamma}\right]}\right],$$

$$\lambda_X^{-\frac{1}{\gamma}} = \frac{V_0}{\left(\frac{\exp\left[\left(A+\left(\frac{\rho-r}{\gamma}\right)-r\right)T\right]-1}{\left[\left(A+\left(\frac{\rho-r}{\gamma}\right)-r\right)T\right]} + \mathbb{E}\left[X_T^{1-\gamma}\right]^{\frac{1}{\gamma}}\exp\left[\left(A-r\left(\frac{\gamma-1}{\gamma}\right)\right)T\right]\right)},$$

4. The optimal weight invested on the risky asset satisfies:

$$w_S^* = \frac{1}{\gamma}\left(\frac{\mu-r}{\sigma^2}\right).$$

5. The utility of the optimal strategy, knowing the probability distribution of X_T, is given by:

$$\mathbb{E}\left[\int_0^T e^{-\rho s}u(j\left[\lambda_X\eta_s e^{(\rho-r)s}\right])ds + U\left(X_T J\left[\lambda_X\frac{1}{\mathbb{E}\left[X_T^{1-\gamma}\right]}\eta_T e^{-rT}\right]\right)\right] =$$

$$\frac{\lambda_X^{\frac{\gamma-1}{\gamma}}}{(1-\gamma)}\mathbb{E}\left[\int_0^T e^{-\rho s}\left[\eta_s e^{(\rho-r)s}\right]^{\left(\frac{\gamma-1}{\gamma}\right)}ds + X_T\left(\frac{1}{\mathbb{E}\left[X_T^{1-\gamma}\right]}\eta_T e^{-rT}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}\right].$$

Thus, the utility of the optimal strategy is equal to:

$$\mathcal{U}^R(V_0) = \frac{V_0^{1-\gamma}}{(1-\gamma)}\Psi\left(\rho, \mathbb{E}\left[X_T^{1-\gamma}\right]^{\frac{1}{\gamma}}; r, \mu, \sigma, T, \gamma\right).$$

2.5 Comparison of the three regimes

To illustrate the main differences between the three previous basic cases, we consider the notion of compensating variation to measure the monetary losses due to total ignorance or partial information about shock randomness. The utility loss from not having access to an optimal portfolio is provided by the compensating variation measure. If an investor with risk aversion γ and initial investment V_0 faces an optimal pair of consumption and portfolio (C^*, V^*), his expected utility is $\mathbb{E}\left[\int_0^T e^{-\rho s}u(C_s^*)ds + U\left(X_T V_T^*\right); V_0\right]$. If this investor selects a non optimal pair (C, V), he will get the same expected utility provided that he invests the initial amount $\tilde{V}_0 \geq V_0$. Therefore, in this case, this investor requires a (theoretical) compensation $\tilde{V}_0$ which satisfies:

$$\mathbb{E}\left[\int_0^T e^{-\rho s}u(C_s^*)ds + U\left(X_T V_T^*\right); V_0\right] = \mathbb{E}\left[\int_0^T e^{-\rho s}u(C_s)ds + U\left(X_T V_T\right); \tilde{V}_0\right].$$

The amount $\tilde{V}_0$ is in line with the certainty equivalent concept in expected utility analysis. It can be viewed as an "implied initial investment" necessary to maintain the level of expected utility. The ratio $\tilde{V}_0/V_0$ provides a quantative measure, called the compensating variation, of the monetary utility loss of not having the optimal investment strategies.

As numerical base case, we consider the following parameter values:

$$\mu = 7\%, \sigma = 20\%, r = 2\%, \rho = 4\%, T = 1 \ year$$

We assume that the random shock X_T has a binomial type distribution:

$$\mathbb{P}\left[X_T = 1\right] = 1 - p \text{ and } \mathbb{P}\left[X_T = a\right] = p, \text{ with } 0 < a < 1.$$

The numerical base value of parameter p is equal to 0.5.

2.5.1 Myopic versus perfect insight

The compensating variation between the myopic and the perfect insight cases is given by:

$$\frac{\widehat{V_0^{M,Pi}}}{V_0} = \left(\frac{\mathbb{E}\left[\Psi\left(\rho, X_T^{(1-\gamma)/\gamma}; r, \mu, \sigma, T, \gamma\right)\right]}{\frac{\Phi\left(\rho, \mathbb{E}\left[X_T^{1-\gamma}\right]; r, \mu, \sigma, T, \gamma\right)}{\Phi(\rho, 1; r, \mu, \sigma, T, \gamma)^{1-\gamma}}}\right)^{\frac{1}{1-\gamma}}.$$

Figure (1) illustrates how the compensating variation varies according to the level of the potential loss a, for different relative risk aversions. For relative risk aversion around 1 (corresponding to the risk-neutral case), the compensating variations are weak (its maximum is only about 0.4%). For weak and high relative risk aversions (respectively, $\gamma = 0.01$ and $\gamma = 10$), the compensating variations are very significant (respectively, about 30% and 18% for a potential loss of 50%).

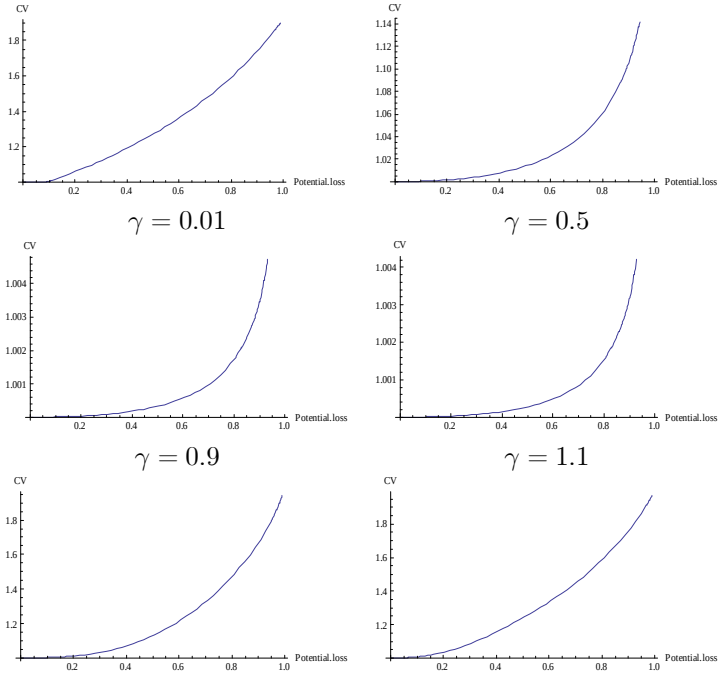

Fig. 1. CV of myopic versus perfectly informed strategies as a function of potential loss

2.5.2 Myopic versus rational expectation

The compensating variation is given by:

$$\frac{\widehat{V_0^{M,R}}}{V_0} = \left(\frac{\Psi\left(\rho, \mathbb{E}\left[X_T^{1-\gamma}\right]^{\frac{1}{\gamma}}; r, \mu, \sigma, T, \gamma\right)}{\frac{\Phi\left(\rho, \mathbb{E}\left[X_T^{1-\gamma}\right]; r, \mu, \sigma, T, \gamma\right)}{\Phi(\rho, 1; r, \mu, \sigma, T, \gamma)^{1-\gamma}}} \right)^{\frac{1}{1-\gamma}}.$$

Figure (2) shows how the compensating variation varies according to the level of the potential loss a, for different relative risk aversions. For relative risk aversion around 1 (corresponding to the risk-neutral case), the compensating variations are weak (its maximum is only about 0.2%). For weak and high relative risk aversions (respectively, $\gamma = 0.01$ and $\gamma = 10$), the compensating variations are significant (about 20% for both risk aversions for a potential loss of 50%). Note that the compensating variations between the myopic and rational expectation cases are smaller than those between the myopic and perfect insight cases.

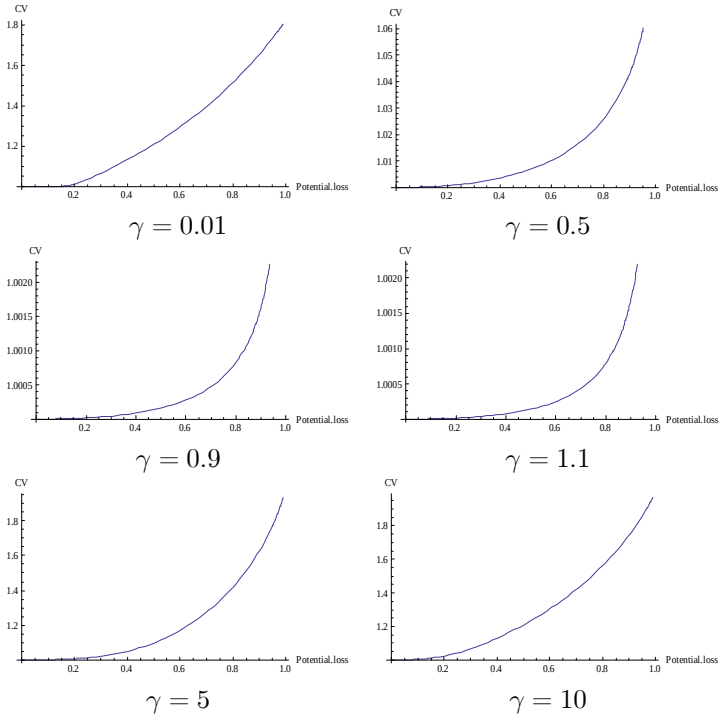

Fig. 2. CV of myopic versus rational expectation strategies as a function of potential loss

2.5.3 Rational expectation versus perfect insight

The compensating variation is given by:

$$\frac{\widehat{V_0^{R,Pi}}}{V_0} = \left(\frac{\mathbb{E}\left[\Psi\left(\rho, (1/X_T)^{\left(\frac{\gamma-1}{\gamma}\right)}; r, \mu, \sigma, T, \gamma\right)\right]}{\Psi\left(\rho, \mathbb{E}\left[X_T^{(1-\gamma)/\gamma}\right]; r, \mu, \sigma, T, \gamma\right)} \right)^{\frac{1}{1-\gamma}}.$$

Figure (3) shows how the compensating variation varies according to different relative risk aversions, for different levels of the potential loss a. For relative risk aversion around 1 (corresponding to the risk-neutral case), the compensating variations are weak (for $\gamma = 1$, the compensating variation is nul). For weak and high relative risk aversions (respectively, $\gamma = 0.01$ and $\gamma = 10$), the compensating variations are significant (about 20% for both risk aversions for a potential loss of 50%). For losses of 30% and 10% due to random shock , the monetary utility losses are always smaller than 10% of the loss due to random shock. Note that the compensating variations between the rational expectation and the perfect insight cases are smaller than those between the myopic and perfect insight cases and also those between the myopic and rational expectation cases.

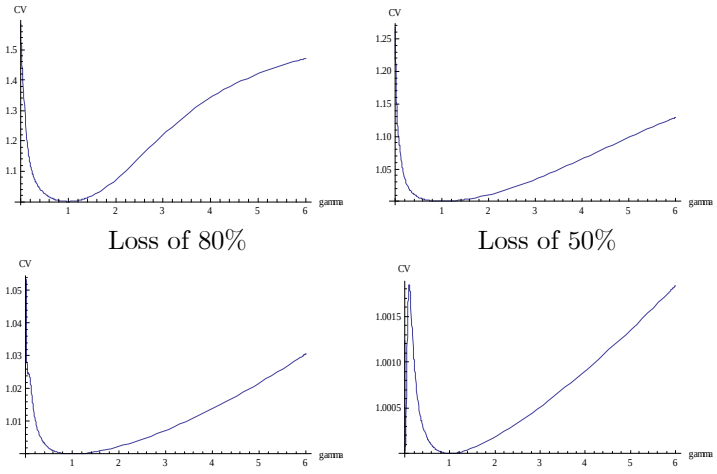

Fig. 3. CV of rational expectation versus perfectly informed strategies as a function of relative risk aversion

3 General dynamic model with a real estate asset

3.1 The financial market

The market is assumed to be arbitrage-free and without friction. Financial transactions occur in continuous-time, along a time period $[0, T]$. Three basic assets are available at any time on the market. (1) An instantaneously riskless *money market* fund, the *Cash,* with a price denoted by C. (2) A *Stock* index fund with a price S. (3) A *Bond* fund denoted by B which is a zero-coupon bond.

To illustrate the results, we assume that the instantaneous riskless interest rate r_t follows an Ornstein-Uhlenbeck process given by:

$$dr_t = a_r(b_r - r_t)dt - \sigma_r dW_{r,t}, \tag{1}$$

where a_r, b_r and σ_r are positive constants and W_r is a standard Brownian motion. The market price of interest rate risk is assumed to be constant (see Vasicek, 1977).

The asset prices dynamics are given by:

(a) Cash:

$$\frac{dM_t}{M_t} = r_t dt. \tag{2}$$

(c) Bond fund:

$$\frac{dB_t}{B_t} = (r_t + \theta_B)dt + \sigma_B dW_{r,t}, \tag{3}$$

(b) Stock index:

$$\frac{dS_t}{S_t} = (r_t + \theta_S)dt + \sigma^S \left(\rho_{SB} dW_{r,t} + \sqrt{(1-\rho_{SB}^2)} dW_{S,t}\right). \tag{4}$$

where W is another standard Brownian motion, independent of W_r, and where the volatilities σ_1, σ_2 and σ_B are positive constants. The parameter θ_S is the constant risk premium of the stock, and θ_B is the risk premium of the bond fund, which is a constant.

(d) Housing price:

$$\frac{dH_t}{H_t} = (r_t + \theta_H - r^{imp})dt + \sigma^H \left(\rho_{HB} dW_{r,t} + \widetilde{\rho}_{HS} dW_{S,t} + \widetilde{\rho}_H dW_{H,t}\right), \tag{5}$$

where (W_r, W_S, W_H) is a standard three-dimensional Brownian motion and r^{imp} denotes the constant imputed rent corresponding to the market value based on the net benefits provided by the house.

Coefficients θ_B, θ_S and θ_H denote the risk premia of bond, stock index and housing prices. Parameters σ_B, σ_S and σ_H correspond to their respective volatilities. The coefficient of correlation between stock and bond is denoted by ρ_{SB}

while ρ_{HB} denotes the coefficient of correlation between housing price and bond. Denote ρ_{SH} the constant correlation between stock and housing prices. Coefficients $\widetilde{\rho}_{HS}$ and $\widetilde{\rho}_{H}$ are defined by:

$$\begin{aligned}\widetilde{\rho}_{HS} &= \frac{\rho_{SH-}\rho_{SB}\rho_{HB}}{\sqrt{(1-\rho_{SB}^2)}},\\ \widetilde{\rho}_{H} &= \sqrt{1-\rho_{HB}^2-\widetilde{\rho}_{HS}^2}.\end{aligned}$$

The investor optimal portfolio weights of the current total wealth V invested on the financial assets M, B and S are respectively denoted by x_M, x_B and x_S. Let c_t denote the consumption rate of perishable goods. Introduce respectively φ_{ot} and φ_{rt} the units of housing owned and rented at time t. Let φ_{C_t} denote the total units of housing occupied ($\varphi_{C_t} = \varphi_{ot} + \varphi_{rt}$).

At any time t, the total wealth is solution of the following stochastic differential equation:

$$\begin{aligned}dV_t &= V_t x_{B,t}\frac{dB_t}{B_t} + V_t x_{S,t}\frac{dS_t}{S_t} + \left[V_t\left(1-x_{B,t}-x_{S,t}\right) - \left(\varphi_{ot}\right)H_t\right] r_t dt \\ &\quad +\varphi_{ot}dH_t - \varphi_{rt}\nu H_t dt - c_t dt.\end{aligned} \tag{6}$$

4 Optimal consumptions and portfolio weights

In what follows, first we recall the model of Kraft and Munk (2011). Then, we analyze the impact of an exogenous factor that may reduce the portfolio value at maturity.

4.1 Optimal consumptions and portfolio weights: the standard case

We consider an investor with an initial capital denoted by V_0. He is assumed to maximize expected utility over the time interval $[0, T]$. Therefore, his consumption and optimal portfolio weights are solutions of the following problem:

$$\underset{(c,\varphi_C,x_B,x_S)}{Max}\ \mathbb{E}\left[\int_0^T U\left(c_t,\varphi_{Ct}\right)dt + \widehat{U}\left(V_T\right)\right].$$

We assume that the utility is time-additive with power specification:

$$U\left(c,\varphi\right) = \frac{\left(c^{\alpha}\varphi^{(1-\alpha)}\right)^{1-\gamma}}{1-\gamma} \text{ and } \widehat{U}\left(V\right) = \frac{V^{1-\gamma}}{1-\gamma} \text{ with } 0<\alpha<1 \text{ and } \gamma\neq 1.$$

At any time t, the utility u index is given by:

$$u_t = \delta\ \mathbb{E}_t\left[\int_t^T e^{-\delta(s-t)}\frac{\left(c_s^{\alpha}\varphi_{Cs}^{(1-\alpha)}\right)^{1-\gamma}}{1-\gamma}ds + \zeta e^{-\delta(T-t)}\frac{{V_T}^{1-\gamma}}{1-\gamma}\right], \tag{7}$$

where $\alpha \in]0,1[$ defines the relative weight of the two consumption goods. Term $c_s^{\alpha}\varphi_{Cs}^{(1-\alpha)}$ corresponds to CES assumption ("constant elasticity of substitution") on both the two consumption goods. The coefficient δ corresponds to the time preference parameter. The coefficient γ denotes the relative risk aversion. The parameter ζ measures the importance of bequest for the individual.

Proposition 1 *The optimal perishable consumption rate and the asset weights are given by:*

$$\begin{aligned}
c_t^*(V_t, r_t, H_t) &= V_t \left[\eta \frac{\alpha\nu}{1-\alpha} \frac{H_t^k}{g(t, r_t, H_t)} \right], \\
\varphi_{C_t}^*(V_t, r_t, H_t) &= V_t \left[\eta \frac{H_t^{k-1}}{g(t, r_t, H_t)} \right], \\
\varphi_{o_t}^*(V_t, r_t, H_t) &= V_t \left[\frac{1}{\gamma} \frac{\xi_I}{\sigma_H} \frac{1}{H_t} + \frac{\partial_r g(t, r_t, H_t)}{g(t, r_t, H_t)} \right], \qquad (8) \\
x_{S,t}^* &= \frac{1}{\gamma} \frac{\xi_S}{\sigma_S}, \\
x_{B,t}^* &= \frac{1}{\gamma} \frac{\xi_B}{\sigma_B} - \frac{\sigma_r}{\sigma_B} \frac{\partial_r g(t, r_t, H_t)}{g(t, r_t, H_t)},
\end{aligned}$$

with $k = (1 - \frac{1}{\gamma})(1-\alpha)$ *and* $\eta = (\delta\alpha)^{\frac{1}{\gamma}} \left(\frac{\alpha\nu}{1-\alpha} \right)^{(k-1)}$.

Proposition 2 *The risk premia* $\lambda_S = \frac{\theta_S}{\sigma_S}, \lambda_B = \frac{\theta_B}{\sigma_B}$ *and* $\lambda_H = \frac{\theta_H}{\sigma_H}$ *are defined from no arbitrage conditions and*
$\lambda'_H = \lambda_H + \left(\nu - r^{imp}\right)/\sigma_H$.
The three parameters ξ_B, ξ_S *and* ξ_I*.are defined from relation:*

$$\begin{aligned}
\xi_B &= \frac{1}{\det} \left[\lambda_B \left(1 - \rho_{SH}^2\right) - \rho_{SB,H}\lambda_S - \rho_{BH,S}\lambda'_H \right] \\
\xi_S &= \frac{1}{\det} \left[\lambda_S \left(1 - \rho_{BH}^2\right) - \rho_{SB,H}\lambda_B - \rho_{SH,B}\lambda'_H \right] \\
\xi_I &= \frac{1}{\det} \left[\lambda'_H \left(1 - \rho_{SB}^2\right) - \rho_{SH,B}\lambda_S - \rho_{BH,S}\lambda_B \right]
\end{aligned}$$

with $\det = 1 + 2\rho_{SB}\rho_{HB}\rho_{SH} - \rho_{SB}^2 - \rho_{HB}^2 - \rho_{SH}^2$ *and we use the notation* $:\rho_{xy,z} = \rho_{xy} - \rho_{x,z} - \rho_{y,z}$.

Proposition 3 *The function* g *is defined by:*

$$\begin{aligned}
&g(t, r_t, H_t) = \\
&\zeta^{\frac{1}{\gamma}} \exp\left[-D_\gamma(T-t) - \frac{\gamma-1}{\gamma} B_\kappa(T-t) r_t \right] \\
&+ \frac{\eta\nu}{1-\alpha} H_t^k \int_t^T \exp\left[-d_1(s-t) - \alpha\frac{\gamma-1}{\gamma} B_\kappa(s-t) r_s \right] ds
\end{aligned}$$

with:

$$\begin{aligned} D_\gamma(\tau) &= \left(\frac{\delta}{\gamma}+\frac{\gamma-1}{2\gamma^2}\widetilde{\lambda}^t\widetilde{\lambda}\right)\tau+\left(\overline{r}+\frac{\gamma-1}{\gamma}\frac{\sigma_r\lambda_B}{\kappa}\right)\frac{\gamma-1}{\gamma}\left(\tau-B_\kappa(\tau)\right) \\ &\quad -\frac{1}{2}\frac{\sigma_r^2}{\kappa^2}\left(\frac{\gamma-1}{\gamma}\right)^2\left(\tau-B_\kappa(\tau)-\frac{\kappa}{2}B_\kappa(\tau)^2\right), \end{aligned}$$

and

$$\begin{aligned} d_1(\tau) &= \left(\frac{\delta}{\gamma}+\frac{\gamma-1}{2\gamma^2}\widetilde{\lambda}^t\widetilde{\lambda}-k\left(\frac{1}{\gamma}\sigma_H\lambda_H'-\nu+\frac{1}{2}(k-1)^2\sigma_H^2\right)\right)\tau \\ &\quad +\alpha\left(\overline{r}+\frac{\gamma-1}{\gamma}\frac{\sigma_r\lambda_B}{\kappa}-\frac{k\sigma_r\sigma_H\rho_{HB}}{\kappa}\right)\frac{\gamma-1}{\gamma}\left(\tau-B_\kappa(\tau)\right) \\ &\quad -\frac{1}{2}\frac{\alpha^2\sigma_r^2}{\kappa^2}\left(\frac{\gamma-1}{\gamma}\right)^2\left(\tau-B_\kappa(\tau)-\frac{\kappa}{2}B_\kappa(\tau)^2\right), \end{aligned}$$

The total optimal wealth satisfies:

$$V_t^* = V_0\mathcal{E}(Z_t) =$$

$$\begin{aligned} &V_0\mathcal{E}(\int_0^t x_{B,t}\frac{dB_s}{B_s}+x_{S,s}\frac{dS_s}{S_s}+\left[\left(1-x_{B,s}-x_{S,s}\right)-\left(\varphi_{os}/V_s\right)H_s\right]r_s ds \\ &+\left(\varphi_{os}/V_s\right)dH_s-\varphi_{rs}\nu H_s ds-\left(c_s/V_s\right)ds) \end{aligned}$$

Proof. See Kraft and Munk (2011). ■

4.2 Optimal solution with an exogenous reduction factor

We assume now that the investor may suffer from losses due to an exogenous factor at maturity. At time T, the investor's wealth may decrease due to an exogenous random shock. This is modelled by a random variable X_T. The random variable X_T is assumed to take its values in $[0,1[$.

4.2.1 Optimal solution with myopic behavior

The individual does not take the shock randomness into account. Thus, his optimal strategies correspond to those of the standard case.

Proposition 4 *The optimal perishable consumption rate and the asset weights*

are given by:

$$\begin{aligned}
c_t^M(V_t, r_t, H_t) &= V_t^M \left[\eta \frac{\alpha\nu}{1-\alpha} \frac{H_t^k}{g(t, r_t, H_t)}\right], \\
\varphi_{C_t}^M(V_t, r_t, H_t) &= V_t^M \left[\eta \frac{H_t^{k-1}}{g(t, r_t, H_t)}\right], \\
\varphi_{o_t}^M(V_t, r_t, H_t) &= V_t^M \left[\frac{1}{\gamma}\frac{\xi_I}{\sigma_H}\frac{1}{H_t} + \frac{\partial_r g(t, r_t, H_t)}{g(t, r_t, H_t)}\right], \\
x_{S,t}^M &= \frac{1}{\gamma}\frac{\xi_S}{\sigma_S}, \\
x_{B,t}^M &= \frac{1}{\gamma}\frac{\xi_B}{\sigma_B} - \frac{\sigma_r}{\sigma_B}\frac{\partial_r g(t, r_t, H_t)}{g(t, r_t, H_t)},
\end{aligned} \tag{9}$$

His indirect utility is given by:

$$\mathbb{E}\left[\int_0^T U\left(c_t^M, \varphi_{Ct}^M\right) dt + \widehat{U}\left(X_T V_T^M\right)\right] =$$

$$\mathbb{E}\left[\int_0^T e^{-\delta t} \frac{\left(c_t^{M\alpha} \varphi_{Ct}^{M(1-\alpha)}\right)^{1-\gamma}}{1-\gamma} dt + \zeta e^{-\delta T} \frac{V_T^{M\,1-\gamma}}{1-\gamma} {X_T}^{1-\gamma}\right].$$

4.2.2 Optimal solution with perfect foresight

The individual maximizes his expected utility in particular with respect to the probability distribution of the random shock X_T.

Proposition 5 *With a given exogenous reduction factor x, the optimal consumptions and portfolio weights are given by:*

$$\begin{aligned}
c_t^{Pi}(V_t, r_t, H_t) &= V_t^{Pi} \left[\eta \frac{\alpha\nu}{1-\alpha} \frac{H_t^k}{\widehat{g}(t, r_t, H_t)}\right], \\
\varphi_{C_t}^{Pi}(V_t, r_t, H_t) &= V_t^{Pi} \left[\eta \frac{H_t^{k-1}}{\widehat{g}(t, r_t, H_t)}\right], \\
\varphi_{o_t}^{Pi}(V_t, r_t, H_t) &= V_t^{Pi} \left[\frac{1}{\gamma}\frac{\xi_I}{\sigma_H}\frac{1}{H_t} + \frac{\partial_r \widehat{g}(t, r_t, H_t)}{\widehat{g}(t, r_t, H_t)}\right], \\
x_{S,t}^{Pi} &= \frac{1}{\gamma}\frac{\xi_S}{\sigma_S}, \\
x_{B,t}^{Pi} &= \frac{1}{\gamma}\frac{\xi_B}{\sigma_B} - \frac{\sigma_r}{\sigma_B}\frac{\partial_r \widehat{g}(t, r_t, H_t)}{\widehat{g}(t, r_t, H_t)},
\end{aligned} \tag{10}$$

where the function $\widehat{g^{Pi}}$ is defined by:

$$\widehat{g^{Pi}}(t, r_t, H_t) = x^{1-\gamma} g(t, r_t, H_t).$$

Proof. Due to the random shock, the new utility on the terminal wealth is defined by:

$$\widehat{U^{Pi}}_x(v) = \frac{(xv)^{1-\gamma}}{1-\gamma}.$$

Then, we apply previous optimization results with this modified utility on the terminal wealth. ■

Corollary 6 *The new perishable good consumption ratio* $c_t^{Pi}(V_t^{Pi}, r_t, H_t)/V_t^{Pi}$ *and housing consumption ratio* $\varphi_{Ct}^{Pi}(V_t^{Pi}, r_t, H_t)/V_t^{Pi}$ *are respectively equal to the corresponding previous ones divided by* $x^{1-\gamma}$*. Thus they are smaller than previous ones for* $\gamma > 1$*. It means that, since the investor takes account of the reduction* x *of his terminal wealth, he reduces his intertemporal consumptions when his relative risk aversion is higher than* 1*. We have:*

$$\frac{\partial_r g(t, r_t, H_t)}{g(t, r_t, H_t)} = \frac{\partial_r \widehat{g^{Pi}}(t, r_t, H_t)}{\widehat{g^{Pi}}(t, r_t, H_t)}.$$

The new ratio $\varphi_{ot}^{Pi}(V_t^{Pi}, r_t, H_t)/V_t^{Pi}$ *of housing owned is equal to previous one and the shares* $x_{B,t}^{Pi}$ *and* $x_{S,t}^{Pi}$ *are not affected by the reduction factor.*

We note that only the consumption and the total units of housing occupied are reduced by the same factor $1/x^{1-\gamma} (< 1,$ *for* $\gamma > 1)$*. The portfolio weights (housing, stock and bond) are not modified.*

His indirect utility is given by:[3]

$$\mathbb{E}\left[\int_0^T U\left(c_t^{Pi}, \varphi_{Ct}^{Pi}\right) dt + \widehat{U}\left(X_T V_T^{Pi}\right)\right] =$$

$$\mathbb{E}\left[\int_0^T e^{-\delta t} \frac{\left(c_t^{Pi\alpha} \varphi_{Ct}^{Pi(1-\alpha)}\right)^{1-\gamma}}{1-\gamma} dt + \zeta e^{-\delta T} \frac{V_T^{Pi 1-\gamma}}{1-\gamma} {X_T}^{1-\gamma}\right].$$

4.2.3 Optimal solution with rational expectation

The individual maximizes his expected utility in particular knowing the exact value x of the random shock X_T. The individual maximizes his expected utility in particular with respect to the probability distribution of the random shock X_T.

Proposition 7 *With an exogenous reduction factor, the optimal consumptions*

[3] It is computed by assuming that the value x is random, since the individual knows x but does not choose himself this latter value.

and portfolio weights are given by:

$$\begin{aligned}
c_t^R(V_t, r_t, H_t) &= V_t^R \left[\eta \frac{\alpha\nu}{1-\alpha} \frac{H_t^k}{\widehat{g}(t, r_t, H_t)} \right], \\
\varphi_{C_t}^R(V_t, r_t, H_t) &= V_t^R \left[\eta \frac{H_t^{k-1}}{\widehat{g}(t, r_t, H_t)} \right], \\
\varphi_{o_t}^R(V_t, r_t, H_t) &= V_t^R \left[\frac{1}{\gamma} \frac{\xi_I}{\sigma_H} \frac{1}{H_t} + \frac{\partial_r \widehat{g}(t, r_t, H_t)}{\widehat{g}(t, r_t, H_t)} \right], \\
x_{S,t}^R &= \frac{1}{\gamma} \frac{\xi_S}{\sigma_S}, \\
x_{B,t}^R &= \frac{1}{\gamma} \frac{\xi_B}{\sigma_B} - \frac{\sigma_r}{\sigma_B} \frac{\partial_r \widehat{g}(t, r_t, H_t)}{\widehat{g}(t, r_t, H_t)},
\end{aligned} \tag{11}$$

where the function $\widehat{g^R}$ *is defined by:*

$$\widehat{g^R}(t, r_t, H_t) = \mathbb{E}\left[X_T^{1-\gamma} \right] g(t, r_t, H_t).$$

Proof. Due to the random shock, the new utility on the terminal wealth is defined by

$$\widehat{U^R}_X(v) = \mathbb{E}\left[\frac{(X_T v)^{1-\gamma}}{1-\gamma} \right].$$

Then, we apply previous optimization results with this modified utility on the terminal wealth. ■

Corollary 8 *The new perishable good consumption ratio* $c_t^R(V_t^R, r_t, H_t)/V_t$ *and housing consumption ratio* $\varphi_{C_t}^R(V_t^R, r_t, H_t)/V_t^R$ *are respectively equal to the corresponding previous ones divided by* $\mathbb{E}\left[X_T^{1-\gamma} \right]$*. Thus they are smaller than previous ones for* $\gamma > 1$*. It means that, since the investor takes account of a potential reduction of his terminal wealth, he reduces his intertemporal consumptions according to his rational expectation, if his relative risk aversion is higher than* 1*. Since we have:*

$$\frac{\partial_r g(t, r_t, H_t)}{g(t, r_t, H_t)} = \frac{\partial_r \widehat{g}(t, r_t, H_t)}{\widehat{g}(t, r_t, H_t)}.$$

The new ratio $\varphi_{ot}^R(V_t^R, r_t, H_t)/V_t^R$ *of housing owned is the and the shares* $x_{B,t}^R$ *is the same as previously and* $x_{S,t}^R$ *are not modified by the reduction factor* $1/\mathbb{E}\left[X_T^{1-\gamma} \right]$*.*

It means that that only the consumption and the total units of housing occupied are reduced by the same factor $1/\mathbb{E}\left[X_T^{1-\gamma} \right] (< 1$*, for* $\gamma > 1)$*. The portfolio weights (housing, stock and bond) remain the same.*

His indirect utility is given by:

$$\mathbb{E}\left[\int_0^T U\left(c_t^R, \varphi_{Ct}^R \right) dt + \widehat{U}\left(X_T V_T^R \right) \right] =$$

$$\mathbb{E}\left[\int_0^T e^{-\delta t}\frac{\left(c_t^{R\alpha}\varphi_{Ct}^{R(1-\alpha)}\right)^{1-\gamma}}{1-\gamma}dt+\zeta e^{-\delta T}\frac{V_T^{R1-\gamma}}{1-\gamma}{X_T}^{1-\gamma}\right].$$

5 Conclusion

Standard portfolio and consumption optimization models do not sufficiently take account of potential additional shock that can occur suddenly and imply significant changes in optimal strategies. Such shock can correspond for instance to loss of employment, divorce or death of one member of the couple. We show how standard optimization solutions are modified by independent and exogenous random shocks. We first consider a purely financial portfolio optimization problem with three basic cases: myopic behavior, perfect foresight and finally rational expectation. For the CRRA utility functions, we determine the explicit optimal solutions. We compute the compensating variations between each pair of solutions to measure the monetary losses due to total ignorance or partial information about shock randomness. Our results are illustrated by a numerical base case that shows the importance of monetary losses when the random shock is not taken into account or not exactly known at the initial date. Then we examine the problem of the determination of optimal housing, consumption and financial investment strategies over the life-time cycle. In the CES and CRRA case, we show how these optimal strategies are modified for the three cases. We provide the exact solutions for each case. In particular, we determine exactly how the potential shock changes or not the optimal strategies. We prove that the financial shares are not affected while the consumption ratios are reduced when the individual has a significant relative risk aversion.

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