Household location, dwelling and tenure types in a dynamic context*

Matthieu de Lapparent[†] André de Palma[‡] Nathalie Picard[§]

March 22, 2011

Abstract

We develop in this article a structural microeconomic model to analyze residential location choices of workers in a dynamic context with perfect information. At the beginning of each period, the decision maker is faced with continuous and discrete decisions: choices of optimal quantity of floor space and consumption level of an outside composite good, and choices of residential location, tenure and dwelling types. At the end of the lifecycle, bequest is left to heirs. We also account for several peculiarities in formulation of the possible intertemporal budget constraints (transaction costs, pay-down, borrowings and savings).

We choose functional forms so that the resulting theoretical model of inter-temporal utility maximization is analytically tractable. We discuss properties of the model and we propose an econometric specification for empirical matters. Our general approach can be formulated as mixtures of Nested Logit probabilistic choice models. We then derive a stylized probabilistic model of residential location choices in relation to road congestion and housing prices for workers who change job locations.

JEL Codes: D11, R21

*We gratefully aknowledge the European Commission for their financial support through grant FP7-SSH-2009-A-244557-SustainCity. Discussions during consortium meetings helped us to improve this paper. We also thank participants of the workshop on equilibrium sorting in urban economics and transport models that was held March, 14-15 2011 at Zürich for their comments and suggestions

[†]Institut Français des Sciences et Technologies des Transports, de l'Aménagement et des Réseaux. Département Economie et Sociologie des Transports. 2 rue de la Butte Verte. F-93166 Noisy-le-Grand Cedex. e-mail: matthieu.de.lapparent@ifsttar.fr

[‡]Ecole Normale Supérieure de Cachan. Centre d'Economie de la Sorbonne. 61 avenue du Président Wilson. 94230 Cachan. France. e-mail: andre.depalma@ens-cachan.fr

[§]Université de Cergy-Pontoise. THéorie Economique, Modélisation et Applications. 33 boulevard du Port. 95011 Cergy-Pontoise Cedex. e-mail: nathalie.picard@eco.u-cergy.fr

1 Introduction

Modelling individual residential location choices is a challenge in several key aspects as it needs to account at least for the large dimension of the set of decisions to be taken, for the dynamics of these decisions, for obvious causal and induced effects on and by transport activity, for market clearing mechanisms, and for very demanding and stringent data requirement. In this article, we develop a structural microeconomic model to analyze residential location choices of workers in a dynamic context with perfect information. We consider a decision maker (person, worker, household head, etc.) living two periods. At the beginning of each period, s/he is faced with continuous and discrete decisions: choices of optimal quantity of floor space and consumption level of an outside composite good, and choices of residential location, tenure and dwelling types.

There is substantial literature on the topic. Referencing all contributions and progress would deserve at least another paper. We however observe that the research topic is often broken into smaller parts and then specialized in relation to a specific problem. Although theoretical and empirical modelling approaches may differ, it is recognized that several determinants have now to be accounted for when dealing with analysis of residential location choices. For instance, the importance of transportation costs has been pointed out by Weisbrod et al. (1980), Anas and Chu (1984), Waddell et al. (2007), Lee and Waddell (2010). They focused on the induced effects of the transportation market on residential location choices. Quigley (1985), Nechyba and Strauss (1998), Brueckner et al. (1999) also focused on the demand for local amenities and neighbourhood in explaining location choice. From the financial perspective, de Palma and Lefevre (1985), Ben-Akiva and de Palma (1986) recognized that transaction costs and moving costs may affect the dynamics of location choices in lenghtening the duration of stay at one location. Brueckner (1997) discussed the dynamics of housing expenditures of homeowners. It also may affect the choice of a tenure type. de Palma et al. (2007) recently showed that existence of capacity constraints in housing supply changes considerably location choices. McFadden (1977), Weisbrod et al. (1980), Thisse (2010), also discussed in a more general way existing tradeoffs that may have consequence on location choices, including differences accross individuals with different socioeconomic characteristics. Analysis of dwelling and tenure choices have also been subsject to several analysis, e.g. Mills (1990), Cho (1997), Skaburskis (1999). They discussed the effects of the attributes of a dwelling type in formulation of individual demand functions.

Addressing simultaneously economic choices of residential location, dwelling and tenure and their dynamics while accounting for interaction with transportation market, with demand for local amenities, and with financial investment constraints is a gap that we fill in the present article. We propose in a first part of the paper a conceptual model in which we include the most relevant determinants of residential location choices: demands for local amenities, financial constraints (pay-down requirement, borrowings and savings), housing prices, income, transportation costs, and moving costs. Indeed, these choices are subject to budget and other technical constraints. In our model, borrowing is allowed in the first period, but not in the second one. We also assume that the interest rate is higher when borrowing for a dwelling than when saving/borrowing on the money market, and that transaction costs apply to real estate. We consider existence of moving costs when changing home location in the second period. Of course, transportation costs affect the choices of workers. Another feature of our model is that we consider existence of a bequest motive. The individual may leave a bequest to heirs at the end of his/her lifecycle for altruistic reasons. The bequest is made of money and/or real estate.

We choose functional forms for utility at each period so that the resulting theoretical model of inter-temporal utility maximization is analytically tractable. The problem is solved in two steps. Firstly, considering as given a series of discrete decisions, we obtain the related indirect utility function. Secondly, the optimal series of discrete decisions is defined as the one that maximizes the indirect utility of the worker. We discuss properties of the model. We perform static comparative analysis and we propose an econometric specification for empirical matters. Our approach is based on random utility maximization (McFadden (1977)) and formulated as mixtures of Nested Logit Random Utility Maximization (see for example Train (2003)). Our theoretical model appears as a building block for detailed analysis of residential location choices.

In the second part of the paper, we develop a stylized probabilistic model of residential location choices in relation to road congestion and housing prices for workers who change job locations. We do not consider choices of dwelling and tenure types. The model is calibrated on the basis of the general model that is derived in the general approach. Considering as given the preferences' parameters, we perform simulation to study how equilibrium of the urban system is changing when key variables (transportation costs, moving costs, etc.) are varying.

2 Model

2.1 Framework

We consider a household living 2 periods $t \in \{1, 2\}$. At each period, it has to choose its home location, its dwelling type, and its tenure type. Because of the potential large size of the choice set, we simplify our approach by assuming that the discrete choices about housing are structured as depicted in figure 1. We consider that the household chooses at each period its home location in a first step, then its dwelling type, and finally its tenure type. We also account for the fact that the choices made in period 1 cause the choices made in period 2. There also may exist feedback effects between the different types of choices and over time.

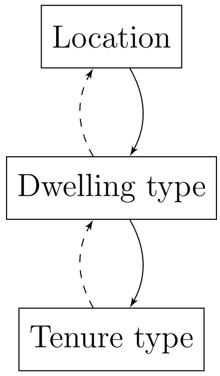

Figure 1: Diagram of discrete choices (plain arrows: causal effects, dashed arrows: feedback effects

We propose a model that complies with this nesting structure in that it should be possible to proceed by backward induction and to study in a first step the dynamics of tenure types given locations and dwelling types before integrating out the other choice dimensions in a second and a third steps.

We thus focus for the moment on the dynamics of tenure types of consumed floor spaces given occupied dwelling types, their locations, and transportation costs. The tree as depicted in figure 2.1 characterizes the full set of series of decisions about tenure type the household could make over its lifecycle. Note that it includes every possible situations that may be empirically observed. Of course, in a deterministic framework, there would be no reason to study all series of choices as, given a structure of housing prices and other exogenous variables, there would be only one possible solution. In reality, we however observe that many individuals deviate from this normative framework. There exists in fact some randomness from the modeller's perspective. That is why we describe totally all possible solutions.

At the beginning of period 1, the household chooses whether to own or to rent a floor space Q_1. At the beginning of period 2, the household chooses first to move out from it and to consume another floor space Q_2 or to stay. In a second step, it chooses its tenure type for this period. It chooses finally to adjust its housing portfolio whenever it would exist and it would be possible to do it by keeping and renting to someone else and/or reselling formerly owned property. We consider that there is no uncertainty. Households have perfect information and they perfectly foresee future prices and resources.

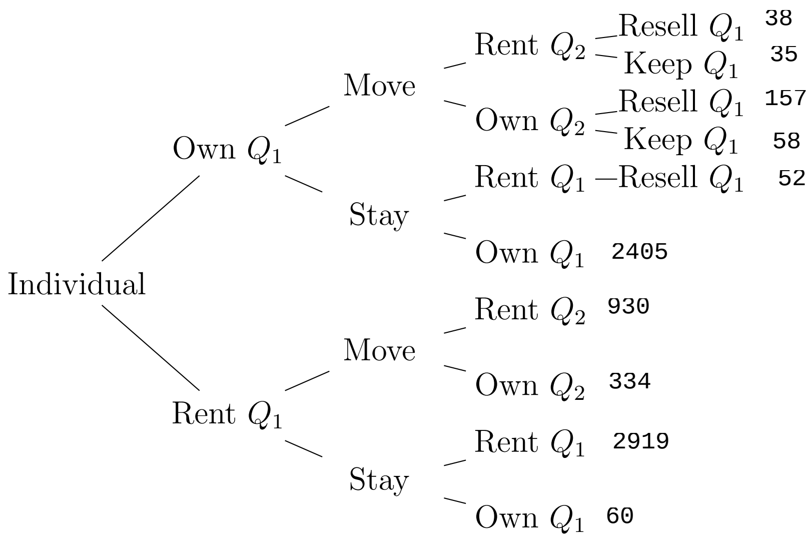

Figure 2: Decision tree for conditional tenure choices

We exclude the possibility to own more than one dwelling that serves or served as home over the lifecycle: buying floor space in period 2 implies to resell the housing quantity bought in period 1. Another point is that we consider existence of a bequest motive. As already stated, leaving a bequest may produce utility for the testator. It then has effects on optimal demands given tenure types over time and, in turn, on the optimal series of tenure types. It is assumed that household can transfer either money or a housing portfolio to its heirs. If the household does not own any dwelling at the end of its lifecycle, the utility that is obtained from leaving a bequest is defined as a function of the strictly positive amount of savings that is left to heirs. If the household owns one housing unit at the end of its lifecycle, it does not leave money to its heirs.

Consider that the household chooses one of the possible trajectory of tenure decisions given dwelling types, locations, and housing portfolio management (refer to figure 2.1). Given prices and budget resources, its problem is to determine its optimal demands for floor spaces Q_1 and Q_2 and other consumption expenditures C_1, C_2 plus a potential level of savings S_2 due to bequest behaviour so as to maximize an intertemporal utility function subject to budget constraints. Dwellings can be held over time in a portfolio. We assume that there is no liquidity constraint at the beginning of the first period in that it is possible to borrow any ammount of money but we assume that the borrowing interest rate is larger than the interest rate of savings at the same period. We also assume that $S_2 \geq 0$, i.e. it is not possible to borrow money at the beginning of the second period. Consider now that the household is able to compute optimal demands and derived indirect utility function for each possible trajectory of decisions. It would then compares the levels of utility of every possible series of decisions and it selects the one that maximizes its utility. As modellers, we want to define more precisely these optimal demands and indirect utility functions so as to develop a structural framework.

2.2 A microeconomic model of conditional tenure choices

We focus in this subsection on the dynamics of tenure type given locations and dwelling types, considering that tenure type restricts to 2 possible situations: owning or renting. The utility a household obtains over its lifecycle, when measured at the beginning of the first period, is defined as a discounted sum of per-period streams of utility.

$$U\left(x_1, C_1, Q_1, x_2, C_2, Q_2, B\right) = u_1\left(x_1, C_1, Q_1\right) + \beta u_2\left(x_2, C_2, Q_2, B\right) \quad (1)$$

where x_1 is the location of the consumed housing quantity during period 1, C_1 is the level of consumption of the outside good during period 1, Q_1 is the housing quantity that is consumed during period 1, x_2 is the location of the consumed housing quantity during period 2, C_2 is the level of consumption of the outside good during period 2, Q_2 is the housing quantity that is consumed during period 2, B is the value of a bequest constituted at the beginning of period 2, and $\beta > 0$ is a discounting factor. Utility is increasing with respect to all its arguments. We assume that

$$u_1\left(x_1, C_1, Q_1\right) = \kappa_1\left(x_1\right) + \alpha_1 \ln\left(C_1\right) + \theta_1 \ln\left(Q_1\right), \quad (2)$$

$$u_2\left(x_2, C_2, Q_2, B\right) = \kappa_2\left(x_2\right) + \alpha_2 \ln\left(C_2\right) + \theta_2 \ln\left(Q_2\right) + \gamma_2 \ln\left(B\right), \quad (3)$$

and

$$B = \pi_2^b\left(x_1\right) Q_1 + \pi_2^b\left(x_2\right) Q_2 + S_2. \quad (4)$$

where $\pi_2^b\left(x_1\right)$ is the unit selling price of a housing quantity that is located at x_1 in period 2 and $\pi_2^b\left(x_2\right)$ is the unit purchasing/reselling price of a housing quantity that is located at x_2 in period 2. x_1 is determined at the beginning of period 1 and x_2 is determined at the beginning of period 2.

We also have to pay attention to the fact that the household does not necessarily change its home location over its lifecycle. In such a situation, it chooses its home once for all at the beginning of the first period of its lifecycle. We have therefore $x_2 = x_1$, $Q_2 = Q_1$, and the value of the bequest cannot include any housing quantity that differs from the one chosen at the beginning of the first period. The functional representation of preferences may therefore be written as

$$U(x_1, C_1, Q_1, C_2, B) = u_1(x_1, C_1, Q_1) + \beta u_2(x_1, C_2, Q_1, B) \tag{5}$$

and

$$B = \pi_2^b(x_1) Q_1 + S_2. \tag{6}$$

The household must respect some budget constraint at the beginning of each period. We now develop their general formulations and we discuss how to derive every special cases along with the different series of choices about tenure types. At the beginning of the first period:

$$p_1 C_1 + \rho(1 + \mu_1)\pi_1(x_1) Q_1 + S_1 = R_1 - D_1(x_1, y_1) \tag{7}$$

where p_1 is the unit price of the outside consumption good during period 1 and $\pi_1(x_1)$ is the unit price of living in a housing good located at x_1 at the beginning of period 1. Of course, as it will be elicited later in the paper, the unit price to pay differs along with the type of tenure. To that extent, we will note $\pi_1^b(x_1)$ the unit purchasing price and $\pi_1^r(x_1)$ the unit rental price. For the moment, we do not need to introduce explicitly these notations. $\rho \in [0, 1]$ models the fraction of the house value that is paid down at the beginning of period 1 when purchased. The rest is paid at the beginning of the second period. To that extent, the household contracts a loan. If the household rents, there is no possibility to postpone or to smooth the due payment and therefore $\rho = 1$. μ_1 is a transaction cost that applies when purchasing a housing quantity. It is assumed to be equal to 0 when dwelling is rented. S_1 is the ammount of savings at period 1. It can be either negative or positive. $R_1 > 0$ is the (exogenous) income of period 1. $D_1(x_1, y_1) > 0$ is a transportation cost when located at x_1 and working at y_1 in period 1. Transportation costs are defined withe respect to a job location but this is without loss of generality. y_1 may be defined as a vector of desired destinations to join from location x_1.

The budget constraint for period 2 is a little bit more complex as it has to account for the possibility to purchase a dwelling in the first period by contracting a loan over the two periods. At the beginning of the second period:

$$\begin{aligned} &p_2 C_2 + ((1-\rho)(1+\tau)(1+\mu_1)\pi_1(x_1) + (1+\mu_2)\pi_2(x_1)) Q_1 \\ &+ (1+\mu_2)\pi_2(x_2) Q_2 + S_2 = (1+r) S_1 + R_2 - D_2(x_2, y_2) - \Delta(x_1, x_2) \end{aligned} \tag{8}$$

where p_2 is the unit price of the outside consumption good during period 2, $\pi_2(x_1)$ is the unit price of living in a dwelling located at x_1 during period

2, and $r > 0$ is the rate of interest of money. It is the same whatever we consider lending or borrowing money. μ_2 is a transaction cost when buying a dwelling in period 2. As for μ_1, it is equal to 0 when renting. $\tau > 0$ is the interest rate of the loan that were contracted at period 1 to buy the housing quantity Q_1 located at x_1 during period 1. We assume that $\tau > r$. $S_2 \geq 0$ is the ammount of savings at period 2. It is necessarily positive because, as already stated, we forbid the household to leave debt at the end of its lifecycle. $R_2 > 0$ is the (exogenous) income in period 2. $D_2(x_2, y_2) > 0$ is a transportation cost when located at x_2 and working at y_2 in period 2. Finally, $\Delta(x_1, x_2)$ is a moving cost when changing home location. It is equal to 0 when not changing home.

We can combine equations 7 and 8 to obtain the intertemporal budget constraint:

$$\begin{aligned} & p_1 C_1 + \tfrac{p_2 C_2}{1+r} + \\ & \left(\rho\left(1+\mu_1\right)\pi_1\left(x_1\right) + \tfrac{(1-\rho)(1+\tau)(1+\mu_1)\pi_1(x_1)+(1+\mu_2)\pi_2(x_1)}{1+r}\right) Q_1 + \\ & \tfrac{(1+\mu_2)\pi_2(x_2)Q_2}{1+r} + \tfrac{S_2}{1+r} = R_1 - D_1\left(x_1, y_1\right) + \tfrac{R_2 - D_2(x_2,y_2) - \Delta(x_1,x_2)}{1+r} \end{aligned} \tag{9}$$

As already stated, depending on whether the household is changing home location or not and depending on its time series of tenure types, equations 1 and 9 take different forms. They are obtained by adapting our notations. For instance, when the decision maker rents the same dwelling both periods, we force $\rho = 1$ (it is not possible to postpone or to smooth the total due payment in period 1), $\mu_1 = \mu_2 = 0$ (there is no transaction cost), $\Delta(x_1, x_2) = 0$ (there is no moving cost as he/she does not change home), $\pi_1(x_1) = \pi_1^r(x_1)$, $\pi_2(x_1) = \pi_2^r(x_1)$, and $\pi_2(x_2) = 0$ (because he/she neither buying nor renting another dwelling at another location).

When the decision maker rents a dwelling in period 1 and buys it in period 2, we force $\rho = 1$, $\mu_1 = 0$, $\Delta(x_1, x_2) = 0$, $\pi_1(x_1) = \pi_1^r(x_1)$, $\pi_2(x_1) = \pi_2^b(x_1)$, $\pi_2(x_2) = 0$, and $S_2 = 0$.

When the decision maker rents two dwellings, one in each period: $\rho = 1$, $\mu_1 = \mu_2 = 0$, $\pi_1(x_1) = \pi_1^r(x_1)$, $\pi_2(x_1) = 0$, and $\pi_2(x_2) = \pi_2^r(x_2)$.

When the decision maker chooses to buy a dwelling in period 1, we have to consider 6 situations. If he/she chooses to live in it both periods without reselling it, we fix $\mu_2 = 0$, $\pi_1(x_1) = \pi_1^b(x_1)$, $\pi_2(x_2) = 0$, $\Delta(x_1, x_2) = 0$, and $S_2 = 0$ (there is no money bequest). If he/she chooses to live in it both periods but resell it in period 2 and rents it to the new owner, then $\mu_2 = 0$, $\pi_1(x_1) = \pi_1^b(x_1)$, $\pi_2(x_1) = \pi_2^r(x_1) - \pi_2^b(x_1)$, $\pi_2(x_2) = 0$, $\Delta(x_1, x_2) = 0$. If he/she changes home location, resells his/her dwelling and rents a new one, then $\mu_2 = 0$, $\pi_1(x_1) = \pi_1^b(x_1)$, $\pi_2(x_1) = -\pi_2^b(x_1)$, $\pi_2(x_2) = \pi_2^b(x_2)$. If he/she changes home location, rents his/her former dwelling and rents a new one, then $\mu_2 = 0$, $\pi_1(x_1) = \pi_1^b(x_1)$, $\pi_2(x_1) = -\pi_2^r(x_1)$, $\pi_2(x_2) = \pi_2^r(x_2)$, and $S_2 = 0$. Finally, if he/she changes home location, resells his/her former dwelling and buys a new one, then $\pi_1(x_1) = \pi_1^b(x_1)$, $\pi_2(x_1) = -\pi_2^b(x_1)$, $\pi_2(x_2) = \pi_2^b(x_2)$, and $S_2 = 0$.

We now turn to explicit formulations of optimal demands and indirect utility functions for each of the possible series of choices as it regards tenure type. Before presenting the analytical solutions of the latter, we prefer to defined further notations to make emerge a clear and unified framework of analysis. To this extent, we note:

- $K_1(x_1) = \kappa_1(x_1) + \beta\kappa_2(x_1)$
- $K_2(x_1, x_2) = \kappa_1(x_1) + \beta\kappa_2(x_2)$
- $W_1(x_1, y_1, y_2, r) = R_1 - D_1(x_1, y_1) + \frac{R_2 - D_2(x_1, y_2)}{1+r}$
- $W_2(x_1, x_2, y_1, y_2, r, \Delta) = R_1 - D_1(x_1, y_1) + \frac{R_2 - D_2(x_2, y_2) - \Delta(x_1, x_2)}{1+r}$
- $\Omega = \alpha_1 + \beta\alpha_2 + \theta_1 + \beta\theta_2 + \beta\gamma_2$
- $b(\rho, \mu_1, \tau, r) = \rho(1+\mu_1) + \frac{(1-\rho)(1+\tau)(1+\mu_1)}{1+r}$
- $a(p_1, p_2, r) = \alpha_1 \ln(\alpha_1) + \beta\alpha_2 \ln(\beta\alpha_2) + \alpha_1 \ln(p_1) + \beta\alpha_2 \ln\left(\frac{p_2}{1+r}\right)$
- $\Xi = (\theta_1 + \beta\theta_2)\ln(\theta_1 + \beta\theta_2) + \beta\gamma_2 \ln(\beta\gamma_2)$
- $\Sigma = (\theta_1 + \beta\theta_2 + \beta\gamma_2)\ln(\theta_1 + \beta\theta_2 + \beta\gamma_2)$
- $\Upsilon = \theta_1 \ln(\theta_1) + \beta\theta_2 \ln(\beta\theta_2) + \beta\gamma_2 \ln(\beta\gamma_2)$
- $\Psi = (\theta_1 + \beta\gamma_2)\ln(\theta_1 + \beta\gamma_2) + \beta\theta_2 \ln(\beta\theta_2)$
- $\Gamma = \theta_1 \ln(\theta_1) + (\beta\theta_2 + \beta\gamma_2)\ln(\beta\theta_2 + \beta\gamma_2)$

2.2.1 Rent the same dwelling during the two periods

Consider in a first step that the household is not changing home location over its lifecycle. The utility function writes then as presented in equation 5. If the household is renting over the two periods, the intertemporal budget constraint then writes as

$$p_1 C_1 + \frac{p_2 C_2}{1+r} + \left(\pi_1^r(x_1) + \frac{\pi_2^r(x_1)}{1+r}\right) Q_1 + \frac{S_2}{1+r} = W_1(x_1, y_1, y_2, r) \tag{10}$$

and the value of the bequest to leave to heirs is defined as $B = S_2$. The system of optimal demands may be written as

$$\begin{cases} C_1^\star = \frac{\alpha_1}{\Omega} \frac{W_1(x_1, y_1, y_2, r)}{p_1} \\ C_2^\star = \frac{\beta\alpha_2}{\Omega} \frac{W_1(x_1, y_1, y_2, r)}{\frac{p_2}{1+r}} \\ Q_1^\star = \frac{\theta_1 + \beta\theta_2}{\Omega} \frac{W_1(x_1, y_1, y_2, r)}{\pi_1^r(x_1) + \frac{\pi_2^r(x_1)}{1+r}} \\ S_2^\star = \frac{\beta\gamma_2}{\Omega} \frac{W_1(x_1, y_1, y_2, r)}{\frac{1}{1+r}} \end{cases} \tag{11}$$

and the indirect utility function may then be written as

$$\begin{array}{l} \mathbf{V}_{r_1,r_1} = K_1\left(x_1\right) + \Xi - \Omega\ln\left(\Omega\right) + \Omega\ln\left(W_1\left(x_1,y_1,y_2,r\right)\right) \\ +\beta\gamma_2\ln\left(1+r\right) - a\left(p_1,p_2,r\right) - \left(\theta_1+\beta\theta_2\right)\ln\left(\pi_1^r\left(x_1\right) + \frac{\pi_2^r(x_1)}{1+r}\right) \end{array} \tag{12}$$

2.2.2 Buy a dwelling in period 1 and stay in it during the two periods

In this case, the decision maker buys a dwelling at the beginning of the first period and lives in it during the two periods. The intertemporal budget constraint becomes

$$p_1C_1 + \frac{p_2C_2}{1+r} + b\left(\rho,\mu_1,\tau,r\right)\pi_1^bQ_1 = W_1\left(x_1,y_1,y_2,r\right) \tag{13}$$

and the bequest function is defined as $B = \pi_2^b\left(x_1\right)Q_1$. The system of optimal demands may then be written as

$$\left\{\begin{array}{l} C_1^\star = \frac{\alpha_1}{\Omega}\frac{W_1(x_1,y_1,y_2,r)}{p_1} \\ C_2^\star = \frac{\beta\alpha_2}{\Omega}\frac{W_1(x_1,y_1,y_2,r)}{\frac{p_2}{1+r}} \\ Q_1^\star = \frac{\theta_1+\beta\theta_2+\beta\gamma_2}{\Omega}\frac{W_1(x_1,y_1,y_2,r)}{b(\rho,\mu_1,\tau,r)\pi_1^b(x_1)} \end{array}\right. \tag{14}$$

and the indirect utility function may be written as

$$\begin{array}{l} \mathbf{V}_{b_1,b_1} = K_1\left(x_1\right) + \Sigma - \Omega\ln\left(\Omega\right) + \Omega\ln\left(W_1\left(x_1,y_1,y_2,r\right)\right) \\ +\beta\gamma_2\ln\left(\pi_2^b\left(x_1\right)\right) - a\left(p_1,p_2,r\right) \\ -\left(\theta_1+\beta\theta_2+\beta\gamma_2\right)\ln\left(b\left(\rho,\mu_1,\tau,r\right)\pi_1^b\right) \end{array} \tag{15}$$

Note that the optimal demands and the indirect utility function simplify further if the household pays down the total value of the purchased housing quantity in period 1 (i.e. $\rho = 1$).

2.2.3 Rent a dwelling in period 1 and buy t in period 2

The decision maker rents a dwelling at the beginning of the first period and he/she then buys it at the beginning of the second period. The intertemporal budget constraint becomes

$$p_1C_1 + \frac{p_2C_2}{1+r} + \left(\pi_1^r\left(x_1\right) + \frac{(1+\mu_2)\pi_2^b(x_1)}{1+r}\right)Q_1 = W_1\left(x_1,y_1,y_2,r\right) \tag{16}$$

and the bequest function is defined as $B = \pi_2^b\left(x_1\right)Q_1$. The system of optimal demands may then be written as

$$\left\{\begin{array}{l} C_1^\star = \frac{\alpha_1}{\Omega}\frac{W_1(x_1,y_1,y_2,r)}{p_1} \\ C_2^\star = \frac{\beta\alpha_2}{\Omega}\frac{W_1(x_1,y_1,y_2,r)}{\frac{p_2}{1+r}} \\ Q_1^\star = \frac{\theta_1+\beta\theta_2+\beta\gamma_2}{\Omega}\frac{W_1(x_1,y_1,y_2,r)}{\pi_1^r(x_1)+\frac{(1+\mu_2)\pi_2^b(x_1)}{1+r}} \end{array}\right. \tag{17}$$

and the indirect utility function may be written as

$$\begin{array}{l}\mathbf{V}_{r_1,b_1} = K_1\left(x_1\right) + \Sigma - \Omega\ln\left(\Omega\right) + \Omega\ln\left(W_1\left(x_1, y_1, y_2, r\right)\right) \\ +\beta\gamma_2\ln\left(\pi_2^b\left(x_1\right)\right) - a\left(p_1, p_2, r\right) \\ -\left(\theta_1 + \beta\theta_2 + \beta\gamma_2\right)\ln\left(\pi_1^r\left(x_1\right) + \frac{(1+\mu_2)\pi_2^b(x_1)}{1+r}\right)\end{array} \tag{18}$$

2.2.4 Buy a dwelling in period 1, resell and rent it to new owner in period 2

The decision maker buys a dwelling in the first period, resells it and rents it to the new owner in the second period. The intertemporal budget constraint is defined as

$$\begin{array}{l}p_1C_1 + \frac{p_2C_2}{1+r} + \\ \left(b\left(\rho, \mu_1, \tau, r\right)\pi_1^b\left(x_1\right) - \frac{\pi_2^b(x_1) - \pi_2^r(x_1)}{1+r}\right)Q_1 + \frac{S_2}{1+r} = \\ R_1 - D_1\left(x_1, y_1\right) + \frac{R_2 - D_2(x_2, y_2)}{1+r}\end{array} \tag{19}$$

The system of optimal demands is then

$$\left\{\begin{array}{l}C_1^\star = \frac{\alpha_1}{\Omega}\frac{W_1(x_1,y_1,y_2,r)}{p_1} \\ C_2^\star = \frac{\beta\alpha_2}{\Omega}\frac{W_1(x_1,y_1,y_2,r)}{\frac{p_2}{1+r}} \\ Q_1^\star = \frac{\theta_1+\beta\theta_2}{\Omega}\frac{W_1(x_1,y_1,y_2,r)}{b(\rho,\mu_1,\tau,r)\pi_1^b(x_1) - \frac{\pi_2^b(x_1)-\pi_2^r(x_1)}{1+r}} \\ S_2^\star = \frac{\beta\gamma_2}{\Omega}\frac{W_1(x_1,y_1,y_2,r)}{\frac{1}{1+r}}\end{array}\right. \tag{20}$$

and the indirect utility function writes

$$\begin{array}{l}\mathbf{V}_{b_1,r_1} = K_1\left(x_1\right) + \Xi - \Omega\ln\left(\Omega\right) + \Omega\ln\left(W_1\left(x_1, y_1, y_2, r\right)\right) \\ +\beta\gamma_2\ln\left(1+r\right) - a\left(p_1, p_2, r\right) \\ -\left(\theta_1 + \beta\theta_2\right)\ln\left(b\left(\rho, \mu_1, \tau, r\right)\pi_1^b\left(x_1\right) - \frac{\pi_2^b(x_1) - \pi_2^r(x_1)}{1+r}\right)\end{array} \tag{21}$$

2.2.5 rent two different dwellings, one in each period

Consider now that the household changes its home location in period 2. The utility function takes the form as presented in equation 1. If the households does not keep any housing good for bequest purpose, the intertemporal budget constraint takes the form as presented in equation 9 and the value of the bequest to leave to heirs is defined as $B = S_2$. In case the decision maker rents two different dwellings, one in each period, the intertemporal budget constraint writes as

$$\begin{array}{l}p_1C_1 + \frac{p_2C_2}{1+r} + \pi_1^r\left(x_1\right)Q_1 \\ +\frac{\pi_2^r(x_2)}{1+r} + \frac{S_2}{1+r} = W_2\left(x_1, x_2, y_1, y_2, r, \Delta\right)\end{array} \tag{22}$$

and the system of optimal demands may then be written as

$$\begin{cases} C_1^\star = \frac{\alpha_1}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{p_1} \\ C_2^\star = \frac{\beta\alpha_2}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{\frac{p_2}{1+r}} \\ Q_1^\star = \frac{\theta_1}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{\pi_1^r(x_1)} \\ Q_2^\star = \frac{\beta\theta_2}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{\frac{\pi_2^r(x_2)}{1+r}} \\ S_2^\star = \frac{\beta\gamma_2}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{\frac{1}{1+r}} \end{cases} \tag{23}$$

and the indirect utility function is defined as

$$\begin{array}{l} \mathbf{V}_{r_1,r_2} = K_2(x_1,x_2) + \Upsilon - \Omega\ln(\Omega) + \Omega\ln(W_2(x_1,x_2,y_1,y_2,r,\Delta)) \\ +\beta\gamma_2\ln((1+r)) - a(p_1,p_2,r) \\ -\theta_1\ln(\pi_1^r(x_1)) - \beta\theta_2\ln\left(\frac{\pi_2^r(x_2)}{1+r}\right) \end{array} \tag{24}$$

2.2.6 Buy in period 1, resell and rent another dwelling in period 2

When the decision maker buys a housing quantity in period 1 and then resells it and rents a dwelling at another location in period 2, the intertemporal budget constraint is defined as

$$\begin{array}{l} p_1C_1 + \frac{p_2C_2}{1+r} + \\ \left(b(\rho,\mu_1,\tau,r)\pi_1^b(x_1) - \frac{\pi_2^b(x_1)}{1+r}\right)Q_1 \\ +\frac{\pi_2^r(x_2)}{1+r} + \frac{S_2}{1+r} = W_2(x_1,x_2,y_1,y_2,r,\Delta) \end{array} \tag{25}$$

and the system of optimal demands may then be written as

$$\begin{cases} C_1^\star = \frac{\alpha_1}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{p_1} \\ C_2^\star = \frac{\beta\alpha_2}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{\frac{p_2}{1+r}} \\ Q_1^\star = \frac{\theta_1}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{b(\rho,\mu_1,\tau,r)\pi_1^b(x_1)-\frac{\pi_2^b(x_1)}{1+r}} \\ Q_2^\star = \frac{\beta\theta_2}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{\frac{\pi_2^r(x_2)}{1+r}} \\ S_2^\star = \frac{\beta\gamma_2}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{\frac{1}{1+r}} \end{cases} \tag{26}$$

and the indirect utility function is defined as

$$\begin{array}{l} \mathbf{V}_{b_1,\text{resell},r_2} = K_2(x_1,x_2) + \Upsilon - \Omega\ln(\Omega) + \Omega\ln(W_2(x_1,x_2,y_1,y_2,r,\Delta)) \\ +\beta\gamma_2\ln((1+r)) - a(p_1,p_2,r) \\ -\theta_1\ln\left(b(\rho,\mu_1,\tau,r)\pi_1^b(x_1) - \frac{\pi_2^b(x_1)}{1+r}\right) \\ -\beta\theta_2\ln\left(\frac{\pi_2^r(x_2)}{1+r}\right) \end{array} \tag{27}$$

2.2.7 Buy in period 1, change home location, rent former dwelling to someone else and rent another dwelling to live in period 2

When the decision maker buys a dwelling in period 1, rents it to someone else in period 2 and rents for himself/herself a dwelling at another location, the value of the bequest to leave to its heirs is defined as $B = \pi_2^b(x_1) Q_1$. The intertemporal budget constraint is defined as

$$\begin{array}{l} p_1 C_1 + \frac{p_2 C_2}{1+r} + \\ \left(b(\rho, \mu_1, \tau, r) \pi_1^b(x_1) - \frac{\pi_2^r(x_1)}{1+r} \right) Q_1 \\ + \frac{\pi_2^r(x_2)}{1+r} = W_2(x_1, x_2, y_1, y_2, r, \Delta) \end{array} \tag{28}$$

and the system of optimal demands may then be written as

$$\begin{cases} C_1^\star = \frac{\alpha_1}{\Omega} \frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{p_1} \\ C_2^\star = \frac{\beta\alpha_2}{\Omega} \frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{\frac{p_2}{1+r}} \\ Q_1^\star = \frac{\theta_1+\beta\gamma_2}{\Omega} \frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{b(\rho,\mu_1,\tau,r)\pi_1^b(x_1) - \frac{\pi_2^r(x_1)}{1+r}} \\ Q_2^\star = \frac{\beta\theta_2}{\Omega} \frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{\frac{\pi_2^r(x_2)}{1+r}} \end{cases} \tag{29}$$

The associated indirect utility function is defined as

$$\begin{array}{l} \mathbf{V}_{b_1,\mathrm{rent},r_2} = K_2(x_1, x_2) + \Psi - \Omega \ln(\Omega) + \Omega \ln(W_2(x_1, x_2, y_1, y_2, r, \Delta)) \\ +\beta\gamma_2 \ln\left(\pi_2^b(x_1)\right) - a(p_1, p_2, r) \\ -(\theta_1 + \beta\gamma_2) \ln\left(b(\rho, \mu_1, \tau, r) \pi_1^b(x_1) - \frac{\pi_2^r(x_1)}{1+r} \right) \\ -\beta\theta_2 \ln\left(\frac{\pi_2^r(x_2)}{1+r} \right) \end{array} \tag{30}$$

2.2.8 Rent in period 1, change home location and buy dwelling in period 2

We now consider the situation where the decision maker rents a housing quantity in period 1 and buys a housing quantity at another location in period 2. In this case, the value of the bequest to leave to heirs is defined as $B = \pi_2^b(x_2) Q_2$ The intertemporal budget constraint takes the following form:

$$\begin{array}{l} p_1 C_1 + \frac{p_2 C_2}{1+r} + \\ \pi_1^r(x_1) Q_1 + \frac{(1+\mu_2)\pi_2^b(x_2)}{1+r} = W_2(x_1, x_2, y_1, y_2, r, \Delta) \end{array} \tag{31}$$

and the system of optimal demands may then be written as

$$\left\{\begin{array}{l} C_1^\star = \frac{\alpha_1}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{p_1} \\ C_2^\star = \frac{\beta\alpha_2}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{\frac{p_2}{1+r}} \\ Q_1^\star = \frac{\theta_1}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{\pi_1^r(x_1)} \\ Q_2^\star = \frac{\beta\theta_2+\beta\gamma_2}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{\frac{(1+\mu_2)\pi_2^b(x_2)}{1+r}} \end{array}\right. \tag{32}$$

The associated indirect utility function is defined as

$$\begin{array}{l} \mathbf{V}_{r_1,b_2} = K_2(x_1,x_2) + \Gamma - \Omega\ln(\Omega) + \Omega\ln(W_2(x_1,x_2,y_1,y_2,r,\Delta)) \\ +\beta\gamma_2\ln\left(\pi_2^b(x_2)\right) - a(p_1,p_2,r) \\ -\theta_1\ln(\pi_1^r(x_1)) - (\beta\theta_2+\beta\gamma_2)\ln\left(\frac{(1+\mu_2)\pi_2^b(x_2)}{1+r}\right) \end{array} \tag{33}$$

2.2.9 Buy in period 1, resell former and buy another in period 2

We finally have to consider the situation where the decision maker buys a dwelling in the first period, then resells it, changes home location, and buys another dwelling in period 2. The intertemporal budget constraint writes as

$$\begin{array}{l} p_1C_1 + \frac{p_2C_2}{1+r} + \\ \left(b(\rho,\mu_1,\tau,r)\,\pi_1^b(x_1) - \frac{\pi_2^b(x_1)}{1+r}\right)Q_1 \\ +\frac{(1+\mu_2)\pi_2^b(x_2)}{1+r} = W_2(x_1,x_2,y_1,y_2,r,\Delta) \end{array} \tag{34}$$

The system of optimal demand is

$$\left\{\begin{array}{l} C_1^\star = \frac{\alpha_1}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{p_1} \\ C_2^\star = \frac{\beta\alpha_2}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{\frac{p_2}{1+r}} \\ Q_1^\star = \frac{\theta_1}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{b(\rho,\mu_1,\tau,r)\pi_1^b(x_1)-\frac{\pi_2^b(x_1)}{1+r}} \\ Q_2^\star = \frac{\beta\theta_2+\beta\gamma_2}{\Omega}\frac{W_2(x_1,x_2,y_1,y_2,r,\Delta)}{\frac{(1+\mu_2)\pi_2^b(x_2)}{1+r}} \end{array}\right. \tag{35}$$

and the indirect utility function may be written as

$$\begin{array}{l} \mathbf{V}_{b_1,\text{resell},b_2} = K_2(x_1,x_2) + \Gamma - \Omega\ln(\Omega) \\ +\Omega\ln(W_2(x_1,x_2,y_1,y_2,r,\Delta)) \\ +\beta\gamma_2\ln\left(\pi_2^b(x_2)\right) - a(p_1,p_2,r) \\ -\theta_1\ln\left(b(\rho,\mu_1,\tau,r)\,\pi_1^b(x_1) - \frac{\pi_2^b(x_1)}{1+r}\right) \\ -(\beta\theta_2+\beta\gamma_2)\ln\left(\frac{(1+\mu_2)\pi_2^b(x_2)}{1+r}\right) \end{array} \tag{36}$$

2.3 Comparative statics

2.3.1 General results

Any increase in wealth of the decision maker has positive effects on consumption C and floor space demand Q in both periods. It increases all levels of utility.

Any increase in the net present value of transportation costs has negative effects on consumption C and floor space demand Q in both periods. It decreases all levels of utility.

An increase in the interest rate r favours consumption C and floor space demand Q in period 2.

Any increase in housing purchasing prices diminishes floor space consumption when buying a dwelling. The result is symmetric for rental housing prices. So, if one type is increasing at a faster rate than the other then the levels of utility are increasing for the latter type of tenure whereas they are diminishing for the former type of tenure.

2.3.2 Particular results

An increase in moving costs Δ_{x_1,x_2} affects wealth of the decision maker only when considering change in home location. It decreases the level of utility for every series of conditional tenure choices when home location is changing. It favours series of conditional tenure choices for which decision maker sticks to the location that is chosen in period 1.

Because we assume that $\tau > r$, an increase in ρ (given that $\rho < 1$, i.e. given that the decision maker buys a dwelling in period 1) affects negatively the floor space to buy in period 1. It also has a negative effect on the levels of utility where purchase of a dwelling at period 1 is made. It favours rental of a dwelling in period 1. Note that if we had assumed $r > \tau$, it would have favoured purchase of a dwelling in period 1.

All else held equal, when purchasing a dwelling in period 1, an increase in τ diminishes floor space consumption and has a negative effect on the level of utility.

Also, an increase in transaction cost μ_1 in period 1 favours rental in this period and an increase in transaction cost μ_2 in period 2 favours rental in this period. When any is increasing, it diminishes the levels of utility of the utility function that are related to purchase of a dwelling.

2.4 Integrating out other choice dimensions

We have derived indirect utility functions that model the dynamics of tenure types conditional to the dynamics of dwelling types and location choices. Integration of these to latter dimensions is not difficult.

We introduce the choices of dwelling types by disaggregating housing prices with respect to a set of, say M, dwelling types. The definition of a dwelling type is itself an important task, from the very simple "house versus appartment" distinction to more complex patterns. One may for instance consider different types of houses and appartments by distinguishing them according to their dates of construction, whether they have a private garden or balcony, and so on. It depends mainly on the available survey data one would like to use and complimentary sources of information as it regards their prices. In our approach, accounting for different dwelling types is made by disaggregating the housing prices with respect to a considered set of M different housing types. The housing prices we defined in the previous subsection ($\pi_1^j(x_1)$, $\pi_2^j(x_1)$, $\pi_2^j(x_2)$, $j \in \{b, r\}$), have all a new superscript $m \in \{\text{type } 1, \cdots, \text{type } M\}$.

We can proceed analogously to consider the choices of locations. Assume for each period $t \in \{1, 2\}$ that there are L possible locations x_t^l, $l \in \{1, \cdots, L\}$. Consider for each location l that the set of prices may be written as $\pi_1^{m,j}\left(x_1^l\right)$, $\pi_2^{m,j}\left(x_1^l\right)$, $\pi_2^{m,j}\left(x_2^l\right)$. These prices are introduced appropriately in the $9M$ utility functions. Because of the structure of our model, the choices of locations play a role not only through housing prices but also through the transportation costs $D_1\left(x_1^l, y_1\right)$ and $D_2\left(x_2^l, y_2\right)$ and through the baseline utility levels $\kappa_1\left(x_1^l\right)$ and $\kappa_2\left(x_2^l\right)$.

2.5 Optimal behavior

As we assume the nesting structure as depicted in figure 1, the decision maker proceeds by backward induction. For any given series of locations and dwelling types, the decision maker is able to evaluate each of the series that concern tenure types. He/she chooses the series of tenure types that maximizes his/her level of utility. In a second step, for any given series of locations, he/she compares these maximum levels of utility for each possible series of dwelling types. He/she selects the one that maximize these maximum levels. In a last step, he/she finally choose the series of location that gives the maximum of the levels of utility among the optimal conditional dwelling and tenure type choices.

From a mathematical perspective, let $\mathcal{L}$ be the set of the series of locations over the two periods, let $\mathcal{K}$ be the set of the series of dwelling types over the two periods, and let $\mathcal{J}$ be the set of the series of tenure types over the two periods. Let $\mathbf{V}_{l,k,j}$ be the level of the indirect utility function for a choice $(l, k, j) \in \mathcal{L} \times \mathcal{K} \times \mathcal{J}$. For the sake of concise notations, we define $\mathbf{z}$ and $\boldsymbol{\phi}$. $\mathbf{z}$ summarizes every exogenous variables that enter the indirect utility functions. $\boldsymbol{\phi}$ summarizes the parameters that enters the indirect utility functions.

The decision maker chooses the combination of series of residential location choices so as to maximize his/her (indirect) level of utility. In our approach, his/her purpose is then to:

$$\max_{l\in\mathcal{L}}\left[\max_{k\in\mathcal{K}}\left[\max_{j\in\mathcal{J}}\left[\mathbf{V}_{l,k,j}\left(\mathbf{z};\boldsymbol{\phi}\right)\right]\right]\right] \tag{37}$$

Note that the decision maker is not assumed to choose sequentially. The decomposition simply represents nesting patterns and structure of the system.

2.6 Random utility maximization framework

From the empirical perspective, we only observe choices of the decision maker. Even though we postulate they derive from our utility maximizing program, we have to consider the utility functions as random variables and set up in a probabilistic framework of analysis. Indeed, as modellers, we do not observe every relevant variables that drive preferences.

We assume that the indirect utility functions may be written as

$$\mathbf{V}_{l,k,j}\left(\mathbf{w};\boldsymbol{\phi},\boldsymbol{\varepsilon}\right)=\bar{\mathbf{V}}_{l,m,j}\left(\mathbf{w};\boldsymbol{\phi}\right)+\varepsilon_{l,k,j} \tag{38}$$

where

$$\bar{\mathbf{V}}_{l,m,j}\left(\mathbf{w};\boldsymbol{\phi}\right)=\bar{\mathbf{V}}_{l}\left(\mathbf{w};\boldsymbol{\phi}\right)+\bar{\mathbf{V}}_{k|l}\left(\mathbf{w};\boldsymbol{\phi}\right)+\bar{\mathbf{V}}_{j|k,l}\left(\mathbf{w};\boldsymbol{\phi}\right) \tag{39}$$

$\mathbf{w}$ includes observed $\mathbf{z}$ and also characteristics of the decision maker. $\boldsymbol{\varepsilon}$ is a random term that is assumed to be independent from $\mathbf{w}$ and distributed as follows:

$$G\left(\boldsymbol{\varepsilon}\right)=\exp\left(-\sum_{l=1}^{L}\left(\sum_{k=1}^{K}\left(\sum_{j=1}^{J}\exp\left(-\omega_k\varepsilon_{l,k,j}\right)\right)^{\frac{\lambda_l}{\omega_k}}\right)^{\frac{\sigma}{\lambda_l}}\right) \tag{40}$$

The discrete choice model that we obtain is a three-level nested Logit, see for example Train (2003). The choice probabilities may be decomposed as

$$\mathbb{P}_{l,k,j}\left(\mathbf{w};\boldsymbol{\phi}\right)=\mathbb{P}_{l}\left(\mathbf{w};\boldsymbol{\phi}\right)\mathbb{P}_{k|l}\left(\mathbf{w};\boldsymbol{\phi}\right)\mathbb{P}_{j|l,k}\left(\mathbf{w};\boldsymbol{\phi}\right) \tag{41}$$

Using this formulation makes appear the induced effect of choosing a series of tenure types on the choice of a series of dwelling types and the induced effect of choosing a series of dwelling types (and a series of tenure types) on the choice of a series of locations in that:

- the expected maximum utility of a series of tenure types cause the choice of a series of dwelling types;
- the expected maximum utility of a series of dwelling types cause the choice of a series of tenure types.

These expected maximum levels of utility take a particularly convenient closed form. The expected maximum utility of a triple l, k, j may be written as

$$\ln\left(\sum_{l=1}^{L} e^{\sigma\bar{\mathbf{V}}_l(\mathbf{w};\boldsymbol{\phi})+\frac{\sigma}{\lambda_l}\ln\left(\sum_{k=1}^{K} e^{\lambda_l\bar{\mathbf{V}}_{k|l}(\mathbf{w};\boldsymbol{\phi})+\frac{\lambda_l}{\omega_k}\ln\left(\sum_{j=1}^{J} e^{\omega_k\bar{\mathbf{V}}_{j|k,l}(\mathbf{w};\boldsymbol{\phi})}\right)}\right)}\right) \tag{42}$$

and the expected maximum levels of utility of each conditional series of choices are characterized by the “logsum” variables.

Another point in modelling is that observed and unobserved heterogeneity of preferences accross a population of decision makers may also affect the structural parameters of the indirect utility functions. It may then be assumed that the parameters of the indirect utility function are themselves defined as function of the observed and unobserved characteristics of the decision maker. In a parametric formulation of this problem, $\boldsymbol{\phi}$ may be drawn from a distribution $h(\boldsymbol{\phi}|\mathbf{w};\boldsymbol{\varphi})$. Our focus is on the unconditional choice probabilities. They appear as continuous mixtures of three-level nested Logit models:

$$\mathbb{P}_{j|l,k}(\mathbf{w};\boldsymbol{\varphi}) = \int_{\mathbb{D}(\boldsymbol{\phi})} \mathbb{P}_{l,k,j}(\mathbf{w};\boldsymbol{\phi})\, h(\boldsymbol{\phi}|\mathbf{w};\boldsymbol{\varphi})\, \mathrm{d}\boldsymbol{\phi} \tag{43}$$

2.7 Data requirements

Empirical implementation of our theoretical model is data demanding in that we need to have available at least longitudinal disaggregate data but not only. Panel data report observed choices of individuals over periods. They also detail socioeconomic and demographic characteristics of these decision makers. They however do not give any information about the attributes of the unchosen alternatives. furthermore, it appears that some of the attributes of all the likely alternatives, especially as it regards local amenities, are not described. We therefore need to complement these data by drawing statistical information in other data sources. As it regards our problem, we see mainly two demanding requirements: information about prices by location, dwelling and tenure types, and information about local amenities by location.

Our empirical model under development is based on the French National 2006 housing survey, in which individuals are also asked to trace back to 2002 as it concerns their housing choices. We will focus on the population of inhabitants of the Paris region. We will complement this dataset by using côtes Callon that provides historical statistical information about housing prices at the “commune” level (town level) by location, dwelling and tenure types. We will further complement our data by drawing additional information on local amenities from geographical data sources. Another point is that of the very large choice set. We will use importance sampling of alternative for estimation of our model (Bierlaire et al. (2008), Lee and Waddell (2010)).

3 A stylized model of residential location choice in relation to job location and road congestion

3.1 Setting

We consider a stylized urban system as depicted in figure 3.1. There are two job locations y_1 and y_2 and three home locations x_1, x_2, and x_3. x_1 is the nearest location from y_1 and the farthest from y_2. x_2 is the nearest location from y_2 and the farthest from y_1. x_3 is in between x_1 and x_2. It neither the nearest from y_1 nor y_2. People are working at y_1 in period 1 and at y_2 in period 2. Job relocation is perfectly anticipated. x_2 cannot be chosen in period 1 but only in period 2. Workers who locate their homes at x_3 stick to this location the two periods. We therefore have to consider three possible series of residential location choices: (x_1, x_1), (x_1, x_2), (x_3, x_3). Of course, we consider at each location that housing prices are function of their respective demand in addition to presence of local amenities. As workers also have to travel to join their job locations, we consider five travel links: (x_1, y_1), (x_1, y_2), (x_3, y_1), (x_3, y_2), (x_2, y_2). There is no need to consider (x_2, y_1) as it is assumed that x_2 is not a possible home location in period 1. As formulated, it is also clear that there exists only one route by "home to work" pair. All the travel links are subject to road congestion. Each congestion function is increasing along with the volume of workers that use the route for which it is defined. The total population of workers is N. N_1 is the number of workers who choose (x_1, x_1), N_2 is the number of workers who choose (x_1, x_2) and N_3 is the number of workers who choose (x_3, x_3).

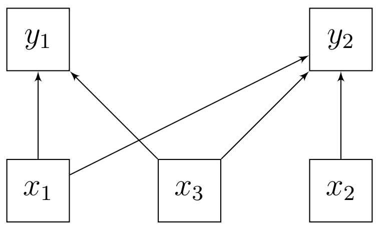

Figure 3: Description of the system

Travel costs are modelled as follows:

$$\begin{cases} D_1(x_1,y_1) = \bar{c}(x_1,y_1) + \delta_0(N_1+N_2) \\ D_2(x_1,y_2) = \bar{c}(x_1,y_2) + \delta_1 N_1 \\ D_2(x_2,y_2) = \bar{c}(x_2,y_2) + \delta_2 N_2 \\ D_1(x_3,y_1) = \bar{c}(x_3,y_1) + \delta_3 N_3 \\ D_2(x_3,y_2) = \bar{c}(x_3,y_2) + \delta_4 N_3 \end{cases} \tag{44}$$

and housing prices are modelled as follows:

$$\begin{cases} \pi_1(x_1) = \bar{\pi}_1(x_1) + \eta_0(N_1+N_2) \\ \pi_2(x_1) = \bar{\pi}_2(x_1) + \eta_1 N_1 \\ \pi_2(x_2) = \bar{\pi}_2(x_2) + \eta_2 N_2 \\ \pi_1(x_3) = \bar{\pi}_1(x_3) + \eta_3 N_3 \\ \pi_2(x_3) = \bar{\pi}_2(x_3) + \eta_4 N_3 \end{cases} \tag{45}$$

To simplify further, we also consider that consider that:

- there is neither choice of a tenure type nor choice of a dwelling type: only renting is allowed and only type of dwelling is available;
- there is no other mode of transport except driving alone a car.

Workers' behaviours is modelled as presented in the former section. As modellers, we set in a random utility maximization framework. Under our assumption, the functional forms of the indirect utility functions limit therefore, after removing their common components because only matters difference in utility levels and after introducing the fact that transportation costs and housing prices are function of the distribution of workers over the series of residential location choices, to three cases. The indirect utility level for worker choosing (x_1, x_1) may be written as

$$\begin{aligned} &\mathbf{V}_{r_1,r_1} = K_1(x_1) + \Xi + \Omega \ln(W_1(x_1,y_1,y_2,r,N_1,N_2)) \\ &- (\theta_1 + \beta\theta_2) \ln\left(\pi_1^r(x_1, N_1+N_2) + \tfrac{\pi_2^r(x_1,N_1)}{1+r}\right) + \epsilon_1 \\ &= \bar{\mathbf{V}}_{r_1,r_1}(\mathbf{s}, N_1, N_2; \boldsymbol{\phi}) + \epsilon_1 \end{aligned} \tag{46}$$

The indirect utility level for worker who chooses (x_1, x_2) may be written as

$$\begin{aligned} &\mathbf{V}_{r_1,r_2} = K_2(x_1,x_2) + \Upsilon + \Omega \ln(W_2(x_1,x_2,y_1,y_2,r,N_1,N_2\Delta)) \\ &-\theta_1 \ln(\pi_1^r(x_1,N_1+N_2)) - \beta\theta_2 \ln\left(\tfrac{\pi_2^r(x_2,N_2)}{1+r}\right) + \epsilon_2 \\ &= \bar{\mathbf{V}}_{r_1,r_2}(\mathbf{s}, N_1, N_2; \boldsymbol{\phi}) + \epsilon_2 \end{aligned} \tag{47}$$

and the indirect utility level for worker who chooses (x_3, x_3) may be written as

$$\begin{aligned} &\mathbf{V}_{r_3,r_3} = K_1(x_3) + \Xi + \Omega \ln(W_1(x_3,y_1,y_2,r,N_3)) \\ &- (\theta_1 + \beta\theta_2) \ln\left(\pi_1^r(x_3,N_3) + \tfrac{\pi_2^r(x_3,N_3)}{1+r}\right) + \epsilon_3 \\ &= \bar{\mathbf{V}}_{r_3,r_3}(\mathbf{s}, N_3; \boldsymbol{\phi}) + \epsilon_3 \end{aligned} \tag{48}$$

We assume that

$$\epsilon_j \overset{iid}{\to} G(\epsilon_j) = \exp\left(-\exp\left(-\frac{\epsilon_j}{\sigma}\right)\right) \tag{49}$$

where σ is the scale of unobserved heterogeneity of preferences. It results that worker's demands for the three series of locations take the form of Multinomial Logit choice probabilities. The probability that worker chooses (x_1, x_1) is defined as

$$\mathbb{P}(x_1, x_1|\mathbf{s}, N_1, N_2, N_3; \boldsymbol{\phi}) = \frac{\exp\left(\frac{\bar{\mathbf{V}}_{r_1,r_1}(\mathbf{s},N_1,N_2;\boldsymbol{\phi})}{\sigma}\right)}{\exp\left(\frac{\bar{\mathbf{V}}_{r_1,r_1}(\mathbf{s},N_1,N_2;\boldsymbol{\phi})}{\sigma}\right)+\exp\left(\frac{\bar{\mathbf{V}}_{r_1,r_2}(\mathbf{s},N_1,N_2;\boldsymbol{\phi})}{\sigma}\right)+\exp\left(\frac{\bar{\mathbf{V}}_{r_3,r_3}(\mathbf{s},N_3;\boldsymbol{\phi})}{\sigma}\right)} \tag{50}$$

The probability that he/she chooses (x_1, x_2) is defined as

$$\mathbb{P}(x_1, x_2|\mathbf{s}, N_1, N_2, N_3; \boldsymbol{\phi}) = \frac{\exp\left(\frac{\bar{\mathbf{V}}_{r_1,r_2}(\mathbf{s},N_1,N_2;\boldsymbol{\phi})}{\sigma}\right)}{\exp\left(\frac{\bar{\mathbf{V}}_{r_1,r_1}(\mathbf{s},N_1,N_2;\boldsymbol{\phi})}{\sigma}\right)+\exp\left(\frac{\bar{\mathbf{V}}_{r_1,r_2}(\mathbf{s},N_1,N_2;\boldsymbol{\phi})}{\sigma}\right)+\exp\left(\frac{\bar{\mathbf{V}}_{r_3,r_3}(\mathbf{s},N_3;\boldsymbol{\phi})}{\sigma}\right)} \tag{51}$$

and the probability that he/she chooses (x_3, x_3) is defined as

$$\mathbb{P}(x_3, x_3|\mathbf{s}, N_1, N_2, N_3; \boldsymbol{\phi}) = \frac{\exp\left(\frac{\bar{\mathbf{V}}_{r_3,r_3}(\mathbf{s},N_3;\boldsymbol{\phi})}{\sigma}\right)}{\exp\left(\frac{\bar{\mathbf{V}}_{r_1,r_1}(\mathbf{s},N_1,N_2;\boldsymbol{\phi})}{\sigma}\right)+\exp\left(\frac{\bar{\mathbf{V}}_{r_1,r_2}(\mathbf{s},N_1,N_2;\boldsymbol{\phi})}{\sigma}\right)+\exp\left(\frac{\bar{\mathbf{V}}_{r_3,r_3}(\mathbf{s},N_3;\boldsymbol{\phi})}{\sigma}\right)} \tag{52}$$

In order to obtain the equilibrium of the system, we therefore have to solve the following fixed point:

$$\begin{cases} N\mathbb{P}(x_1, x_1|\mathbf{s}, N_1, N_2, N_3; \boldsymbol{\phi}) = N_1 \\ N\mathbb{P}(x_1, x_2|\mathbf{s}, N_1, N_2, N_3; \boldsymbol{\phi}) = N_2 \\ N\mathbb{P}(x_3, x_3|\mathbf{s}, N_1, N_2, N_3; \boldsymbol{\phi}) = N_3 \\ N = N_1 + N_2 + N_3 \end{cases} \tag{53}$$

3.2 Numerical application

3.2.1 Parameters

The parameters we choose for our numerical application are presented in table 1. We consider a urban system without difference in local amenities at the different locations. Housing prices are set with the same mechanism at every locations. There is congestion on transportation links except on (x_2, y_2). Excepted limited capacity on link (x_2, y_2), what differs in transportation, what differs in transportation costs is the distance to be travelled when choosing a particular series of location choices.

3.2.2 Simulations

Tables 2, 3 and 4 report simulation results when we make vary some of the parameters of the model. Table 2 summarizes the incidence on distribution of choices of an increase or a decrease in moving costs Δ. Table 3 reports the results when we make vary the level of dependence of transportation costs to total road demand on link (x_2, y_2). Finally, table 4 reports simulation results when we make vary the dependence of housing proces to total housing demand at x_2. These simulations show how the different tradeoffs between housing prices, transportation costs and moving costs, determine the equilibrium of the postulated urban system.

Moving costs play a key role in choice of a series of locations. The larger the moving cost, the lesser the probability to choose (x_1, x_2), and vice-versa. An increase in moving costs Δ benefits naturally to the probability to choose (x_1, x_1) or (x_3, x_3). The probability to choose (x_3, x_3) is also the less affected by a decrease in moving costs Δ even though it is the series of choices with the the largest transportation costs when compared to (x_1, x_1). But the way housing prices set at x_1 compensates these additional costs. The results are similar when considering a variation of capacity of the link (x_2, y_2). The larger this capacity, the larger the probability to choose (x_1, x_2), and vice-versa. When road capacity decreases, workers are more likely to choose (x_3, x_3). We finally see that higher pressure on housing prices at x_2 in period 2 naturally favours the choice of (x_3, x_3) because of higher pressure on housing prices at x_1 in period 1 and 2 that compensates the gain in transportation costs when choosing (x_1, x_1).

4 Conlusion

We developed a structural microeconomic framework of analysis to analyze simultaneously the dynamics of residential location choices in several aspects: location, dwelling, and tenure. We accounted for realistic and adapted intertemporal budget constraints while allowing for a bequest behaviour. Our analytical formulation making it tractable for empirical matters, we therefore proposed an econometric formulation of the approach by formulating a mixture of nested-Logit probabilistic choice models. We discussed demanding and stringent data requirements to implement it.

Our derived stylized probabilistic choice model shed light on how dynamics of residential location may be affected by a planned change in job location while accounting for road congestion and pressure on housing prices. Our simulations pointed out the mechanisms of equilibrium of the dynamics of location choices at stake.

Our work may however be further continued in several ways. Firstly, the model may be extended to a $T > 2$ periods inter-temporal maximization program. Secondly, the assumption about perfect information and perfect foresight of market variables has to be called into question. The approach may be formulated as a dynamic discrete choice model with forward-looking economic agents. Thirdly, even though data requirements are rather sizeable and stringent, our proposed econometric formulation needs to be estimated and tested to conclude on whether it is a sensible approach. As it regards our stylized model, it may be further developed to account for choices of tenure and dwelling types. We also think that using estimated parameters from the econometric model would give a better basis to perform simulation of an urban system equilibrium.

References

Anas, A. and Chu, C. (1984). Discrete choice models and the housing price and travel to work elasticities of location demand. *Journal of Urban Economics*, 15(1):107 – 123.

Ben-Akiva, M. and de Palma, A. (1986). Analysis of a dynamic residential location choice model with transaction costs. *Journal of Regional Science*, 26(2):321–341.

Bierlaire, M., Bolduc, D., and McFadden, D. (2008). The estimation of generalized extreme value models from choice-based samples. *Transportation Research Part B: Methodological*, 42(4):381–394.

Brueckner, J. (1997). Consumption and investment motives and the portfolio choices of homeowners. *The Journal of Real Estate Finance and Economics*, 15(2):159–180.

Brueckner, J., Thisse, J., and Zenou, Y. (1999). Why is central Paris rich and downtown Detroit poor? An amenity-based theory. *European Economic Review*, 43:91–107.

Cho, C. (1997). Joint choice of tenure and dwelling type: a multinomial Logit analysis for the city of Chongju. *Urban Studies*, 34(9):1459–1473.

de Palma, A. and Lefevre, C. (1985). Residential change and economic choice behavior. *Regional Science and Urban Economics*, 15(3):421 – 434.

de Palma, A., Picard, N., and Waddell, P. (2007). Discrete choice models with capacity constraints: An empirical analysis of the housing market of the greater paris region. *Journal of Urban Economics*, 62(2):204–230.

Lee, B. and Waddell, P. (2010). Residential mobility and location choice: a nested logit model with sampling of alternatives. *Transportation*, 37(4):587–601.

McFadden, D. (1977). Modelling the choice of residential location. Cowles Foundation Discussion Papers 477, Cowles Foundation for Research in Economics, Yale University.

Mills, E. (1990). Housing tenure choice. *The Journal of Real Estate Finance and Economics*, 3(4):323–331.

Nechyba, T. J. and Strauss, R. P. (1998). Community choice and local public services: A discrete choice approach. *Regional Science and Urban Economics*, 28(1):51–73.

Quigley, J. M. (1985). Consumer choice of dwelling, neighborhood and public services. *Regional Science and Urban Economics*, 15(1):41 – 63.

Skaburskis, A. (1999). Modelling the choice of tenure and building type. *Urban Studies*, 36(13):2199–2215.

Thisse, J. (2010). Toward a unified theory of economic geography and urban economics. *Journal of Regional Science*, 50:281–296.

Train, K. E. (2003). *Discrete Choice Methods with Simulation*. Number 9780521816960 in Cambridge Books. Cambridge University Press.

Waddell, P., Ulfarsson, G. F., Franklin, J. P., and Lobb, J. (2007). Incorporating land use in metropolitan transportation planning. *Transportation Research Part A: Policy and Practice*, 41(5):382–410.

Weisbrod, G., Lerman, S., and Ben-Akiva, M. (1980). Tradeoffs in residential location decisions: transportation vs. other factors. *Transportation Policy and Decision Making*, 1:13–26.

Parameter	Label	Value
N	Size of the population	1
Δ	Moving cost	0.2
r	Interest rate of money	0.015
β	Discount factor	0.5
α_1	Weight of consumption C in period 1	0.4
α_2	Weight of consumption C in period 2	0.4
β_1	Weight of floor space Q in period 1	0.2
β_2	Weight of floor space Q in period 2	0.2
γ_2	Weight of bequest B in period 2	0.1
R_1	Income in period 1	1
R_2	Income in period 2	1
$\kappa_1(x_1)$	Utility of amenities at x_1 in period 1	0.1
$\kappa_2(x_1)$	Utility of amenities at x_1 in period 2	0.1
$\kappa_2(x_2)$	Utility of amenities at x_2 in period 2	0.1
$\kappa_1(x_3)$	Utility of amenities at x_3 in period 1	0.1
$\kappa_2(x_3)$	Utility of amenities at x_3 in period 2	0.1
$\bar{\pi}_1(x_1)$	Fixed housing price at (x_1) in period 1	$0.5\kappa_1(x_1)$
$\bar{\pi}_2(x_1)$	Fixed housing price at (x_1) in period 2	$0.5\kappa_2(x_1)$
$\bar{\pi}_2(x_2)$	Fixed housing price at (x_2) in period 2	$0.5\kappa_2(x_2)$
$\bar{\pi}_1(x_3)$	Fixed housing price at (x_3) in period 1	$0.5\kappa_1(x_3)$
$\bar{\pi}_2(x_3)$	Fixed housing price at (x_3) in period 2	$0.5\kappa_2(x_3)$
η_0	Dependence of housing prices to total demand at x_1 in period 1	0.1
η_1	Dependence of housing prices to total demand at x_1 in period 2	0.1
η_2	Dependence of housing prices to total demand at x_2 in period 2	0.1
η_3	Dependence of housing prices to total demand at x_3 in period 1	0.1
η_4	Dependence of housing prices to total demand at x_3 in period 2	0.1
$\bar{c}(x_1, y_1)$	Fixed transportation cost on link (x_1, y_1)	0.1
$\bar{c}(x_1, y_2)$	Fixed transportation cost on link (x_1, y_2)	0.17
$\bar{c}(x_2, y_2)$	Fixed transportation cost on link (x_2, y_2)	0.1
$\bar{c}(x_3, y_1)$	Fixed transportation cost on link (x_3, y_1)	0.14
$\bar{c}(x_3, y_2)$	Fixed transportation cost on link (x_3, y_2)	0.14
δ_0	Dependence of transportation cost to total travel on link (x_1, y_1)	0
δ_1	Dependence of transportation cost to total travel on link (x_1, y_2)	0
δ_2	Dependence of transportation cost to total travel on link (x_2, y_2)	0.1
δ_3	Dependence of transportation cost to total travel on link (x_3, y_1)	0
δ_4	Dependence of transportation cost to total travel on link (x_3, y_2)	0
σ	scale of preferences' heterogeneity	1

Table 1: Parameters for numerical application

	Base	$\Delta + 50\%$	$\Delta - 50\%$
N_1	0.341	0.353	0.331
N_2	0.295	0.272	0.315
N_3	0.364	0.375	0.354

Table 2: Distribution of choices as function of moving costs

	Base	$\delta_2 + 100\%$	$\delta_2 - 100\%$
N_1	0.341	0.344	0.338
N_2	0.295	0.289	0.301
N_3	0.364	0.367	0.361

Table 3: Distribution of choices as function of total road demand on (x_2, y_2)

	Base	$\eta_2 + 100\%$	$\eta_2 - 100\%$
N_1	0.341	0.344	0.336
N_2	0.295	0.289	0.304
N_3	0.364	0.367	0.360

Table 4: Distribution of choices as function of total demand for housing at x_2